

Final Exam, April 28 — 2 pages, 5 questions. All sub-questions carry equal weight.

1. (20%) Consider the ARMA(2,2) model

$$y_t = 3 + 0.4y_{t-1} + 0.6y_{t-2} + e_t - e_{t-1} ,$$

where e_t is white noise with variance 3.

- i) Assume $y_0 = 10$, $e_0 = 1$, $e_1 = 0$, and $y_{-1} = 4$ find the mean of y_2, y_3 and y_4 conditional on all variables indexed $t = 1$ or earlier (as given).
- ii) And what is the variance of y_2, y_3 and y_4 conditional on conditional on all variables indexed $t = 1$ or earlier?
- iii) Is this process (model) stable?

2. (25%) Consider the case of the 2 agents, Jones (J) and Smith (S) who live for 3 periods in a 4 states-of-the-world model where agents can trade using a full set of Arrow securities. Assume each of the agents have utility functions

$$\ln(C_0) + E_0 \ln(C_1) .$$

The following table gives the possible endowments and the probabilities for Jones (J) and Smith (S):

	Smith			Jones		
State of the world:	A	B	C	A	B	C
period 0 endowment	20	20	20	20	20	20
period 1 endowment	10	30	20	30	10	20
Period 0 probability:	.25	.25	.5	.25	.25	.5

Assume that Jones and Smith are the only two agents in the world.

- a) For this question assume that Jones and Smith in period 0 can trade in an Arrow security for state A and an Arrow security for state B but not in any other assets. Find the prices of the Arrow securities (you can still write them as $p/(1+r)$ although the interest rate is not quite meaningful in the absence of a safe asset).
- b) Find the consumption of both agents in each period and state of the world.
- c) From now on assume that there is a full set of Arrow securities. Find the rate of interest between period 0 and period 1.

3. (15%) Assume that the PIH holds and that the discount rate is 10 percent (and therefore also the rate of interest is 10 percent). Assume that an agent initial assets of 0 and has income in period 0 and in all future periods which satisfies the relation $Y_t = 10 + 0.5^t$.

Find the agent's consumption C_t in all periods.

4. (20%) Derive the consumption-CAPM as in class.

5. (20%) A consumer lives for 2 periods and earns $Y_1 = 20\$$ in period 1, and in period 2 he or she earns X where X is normally distributed with mean 20 and variance 2. The consumer starts with 0 assets and maximizes

$$U(C_1) + \frac{1}{1.10} E_1 U(C_2) ,$$

where

$$U(C) = 1 - \exp(-C) .$$

Assume that the safe rate of interest is 10 percent.

a) Let B denote the amount lent in period 1 (or, equivalently, the amount of a safe bond bought). Assuming that the agent doesn't have access to any other assets, find B and the consumption in each period (for period 2, that means the consumption plan listing the distribution of consumption in period 2.)

b) Now assume that the agent can only trade in a stock (equity). Let the amount of equity bought be S (it can be negative). Assume that the stock has a log-normal gross rate of return R_2^s with mean x and variance $Var\{\log R^s\} = 2$ (the numbers are chosen for simplicity, not to get a realistic return). Assume that you know that $C_1 = 21$ and the agent's consumption in the second period, C_2 , is normally distributed with mean 22 and variance 4 and with covariance $Cov(\log R^s, C_2) = 1$.

Find x .