ECONOMICS 7344, Spring 2013
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Final Exam, April 29 — 3 pages, 5 questions. All sub-questions carry equal weight.

1. (20%) Consider the AR(1) model

$$y_t = 3 - 0.5y_{t-1} + e_t \ ,$$

where e_t is white noise with variance 1. Assume the rate of interest is 50 percent (to simplify algebra). Further, assume that the PIH holds. (To be precise, a fraction λ of income accrues to such consumers.)
i) Find Δc_t as a function of e_t.
ii) What is the variance of Δy_t?
iii) Find the coefficient α in the relation $\Delta c_t = \alpha \, \Delta y_t$.

2. (20%). Consider two countries home and foreign who each produces two goods, one tradeable good and one non-tradeable good neither of which can be stored. Assume the countries maximize

$$U(C_0^T, C_0^N) + E_0\{U(C_1^T, C_1^N)\} \ ,$$

where $U(C_t^T, C_t^N) = \log C_t^T \, \log C_t^N$ and variables with superscript T are tradeable and variables with superscript N are non-tradeable.

Assume the endowments in period 0 are as follows: home is $Y_0^T = 15$, $Y_0^N = 10$ for "home" and $Y_0^{*T} = 15$, $Y_0^{*N} = 10$ for "foreign." In period 1, there are two states of the world, labeled "a" and "b." The endowments in period 1, state a are: $Y_1^{Ta} = 10$, $Y_1^{Na} = 20$ and $Y_1^{*Ta} = 20$, $Y^{*Na_1} = 20$ and in period 1, state b, they are: $Y_1^{Tb} = 20$, $Y_1^{Nb} = 10$ and $Y_1^{*Tb} = 10$, $Y_1^{*Nb} = 10$. Assume Arrow securities exist for all goods in all states and that the probability of each state is 0.5.
i) Find the price of Arrow security "a" for the tradeable goods.
ii) Explain intuitively why the prices of the tradeables are, or are not, actuarially fair.

3. (20%) a) Describe the price setting assumptions in the Fischer model of price rigidity (write down the equations).
b) The solution for output is linear in m_t, $E_{t-1}m_t$, and $E_{t-2}m_t$. What is the coefficient to m_t? Explain the logic of your answer (this can, and should, be done precisely).
c) Explain what point(s) Fischer was making about rational expectations models. (You don't need to mention all the details from the book, but you need to get the main point across.)

4. (40%) Consider the case of the 2 agents, Jones (J) and Smith (S) who live for 3 periods

in a 4 states-of-the-world model where agents can trade using a full set of Arrow securities. Assume each of the agents have utility functions

$$\ln(C_0) + E_0 \ln(C_1) + E_0 \ln C(_2) \ .$$

The following table gives the possible endowments and the probabilities for Jones (J) and Smith (S):

	Smith				Jones			
State of the world:	1	2	3	4	1	2	3	4
period 0 endowment	20	20	20	20	20	20	20	20
period 1 endowment	10	10	30	30	30	30	10	10
period 2 endowment	10	30	10	30	10	30	10	30
Period 0 probability:	.25	.25	.25	.25	.25	.25	.25	.25

To explain the table: State of the world 1 describes the situation where the endowments are (period 0: Smith 20 and Jones 20, period 1: S 10 and J 30, period 2: S 10 and J 10). The probability is the joint probability for output in period 0, 1, and 2, so for example the period 0 probability that (Smith's endowment is 20 in period 0 and 10 in period 1 *and* Jones' endowment is 20 in period 0 and 30 in period 1) is .25+.25=0.5. From these probabilities you can find any marginal and conditional probabilities that you may need.

Assume that Jones and Smith are the only two agents in the world.

a) For this question assume that Jones and Smith in period 0 can trade in a bond that matures in period 1 (equivalently, one can borrow from the other in period 0). In period 1 no debt can be issued. Find the rate of interest in period 0 (the interest rate of a loan issued in period 0 to be paid in period 1) and the level of consumption for each agent.

b) From now on assume that there is a full set of Arrow securities. Find the period 0 price of the following Arrow securities: one security that pays out one unit of the good in period 1 in states of the world 1 and 2 (this is an Arrow security since those states are identical in period 1).

c) And find the price for one that pays out in period 2 in state 1.

d) Find the rate of interest between period 0 and period 1.

e) Find the one period forward rate of interest between period 1 and period 2.

f) Explain intuitively which the forward rate would be negative or positive.