

Final Exam, April 27 — 5 questions. All sub-questions carry equal weight unless otherwise noted.

1. Assume that income follows the ARMA process

$$a(L)y_t = 3 + b(L)e_t$$

where e_t is white noise.

i) (5%) Show (i.e., just write it down) the formula for how consumption reacts to an innovation (e_t) in income if the PIH holds.

ii) (10%) An explicit example would be:

$$y_t = 3 - .9y_{t-1} + e_t - e_{t-1} .$$

For this process, what would be the change in consumption following a 100\$ innovation to income assuming an interest rate of 6%?

2. (15%) Consider the case of an economy with four states-of-the-world. Assume that an asset S_1 exists that pays 8 units in period 1 if state A occurs, 1 unit if state B occurs, and nothing if state C or D occurs. Another asset S_2 exists which pays 1 unit in period 1, if state C occurs, and nothing in states A, B, and D. A third asset S_3 pays 0 units in period 1 if state C occurs, and 1 unit in states A, B, and D. Finally, an asset paying 8 units in period 1 for sure can be traded.

Is the set of assets equivalent to a full set of Arrow securities? (We need to be able to see how you arrive at your answer; i.e., “yes,” or “no” only gives 0 points even if the answer is correct.)

3. (30%) Assume that a representative agent has a utility function

$$U(C, L) = C - \delta L^\gamma ,$$

where δ is a positive parameter and $\gamma > 1$. Assume that agent $i, i = 1, \dots, N$ supplies output Q_i produced by the production technology $Q = L$. The agent sets the relative price P_i/P , where P is the aggregate price index (assume that N is large so a change in P_i doesn't change P) and faces a demand function

$$Q_i = Y \left(\frac{P_i}{P}\right)^{-\eta} .$$

The agent supplies labor L_i to the market at the equilibrium wage rate W and hires labor in the amount of Q_i .

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- a) Find the optimal relative price $\frac{P_i}{P}$ (where the agent takes P as given).
- b) Find the agent's labor supply as a function of the real wage.
- c) Find the equilibrium level of output in the economy.
- d) Does the equilibrium level of output increase or decrease with the parameter δ . Explain the intuitive logic behind this result.
- e) Does the equilibrium level of output increase or decrease with the parameter η . Explain the intuitive logic behind this result.

4. (10%) Explain how the efficient market hypothesis of Fama relates to modern (Euler-equation-based) finance.

5. (30%) Consider the case of a 2 agents ("Home" and "Foreign"), 2 periods, 2 states-of-the-world model where agents can trade using a full set of Arrow securities. Assume that both agents have quadratic utility functions $U(C_0) + E_0U(C_1)$, where $U(C_t) = C - \frac{1}{200}C^2$. Assume that the endowment of the first agent is $y_0 = 3$ and that the endowment of the second agent in period 0 is $y_0^* = X$

The following table gives the possible endowments and the probabilities for Home and Foreign:

State of the world:	Home		Foreign	
	A	B	A	B
period 1 endowment	2	4	2	4
probability:	.5	.5	.5	.5

- a) Assume the prices $\frac{p^S}{1+r}$ of the Arrow-Debreu assets for each of the 2 states of the world are equal to $\frac{1}{2.2}$. What is X ?
- b) Now assume that you don't know the prices but you know that $X = 3$. Find the rate of interest. Explain in economic terms why it is positive or negative.
- c) Assume that now only bonds can be traded. Find the rate of interest.
- d) Explain in economic terms why the rate of interest would change or not change?