ECONOMICS 7344, Spring 2010
Bent E. Sørensen
April 30, 2010

Final Exam, April 30 —5 questions. All sub-questions carry equal weight unless otherwise indicated.

1. (30%) Consider a setup with 2 periods (1 and 2) and 3 states of the world (A, B, and C) in period 2. An agent has income ("output" in Obstfeld-Rogoff) $Y_1 = 10$, $Y_2^A = 15$, $Y_2^B = 10$, and $Y_2^C = 5$. The probabilities are $\pi^S = 1/3$ for $S = A, B$ and C. Assume that the agent can trade in Arrow Securities with price $\frac{p^s}{1+r}$ for state S, where $p^A = p^B = p^C$, $r = 10\%$ and the discount factor is $\beta = \frac{1}{1+r}$. Assume that the agent has utility function $U(C) = C - \frac{a}{2}C^2$ and the agent maximizes $U(C_1) + \beta E_1 U(C_2)$. (Assume a is small enough that marginal utility is positive.)

a) Find C_1, C_2^A, C_2^B, and C_2^C.
b) Explain intuitively why the agent has positive or negative (you need to explain which) borrowing in period 1 (positive "borrowing" means $C_1 > Y_1$)?
c) Now assume $U(C) = \log C$ and explain if the agent now borrows more in period 1 than you found in part b. You have to explain the logic, even if you derive the solution, but you do not have to solve the model if you explain, using concepts from class, what will happen).
d) Now assume that the world consists of the agent described plus another agent with output Y_t^{*S} for $t = 1, 2$ and $S = A, B, C$ that is identical to that of the first agent. (Prices are now endogenous.) Find the rate of interest under the assumptions (log-utility) of part c. (Here you have to assume that $\beta = \frac{1}{1.1}$ while the interest rate is endogenous.)
e) Explain, using terms from class, whether the interest is higher or lower than the time-discount rate of 10 percent. (You will get full points for a correct explanation even if you didn't get the correct number in part d.)
f) Now, keep assuming that there are two agents with the endowments described but with identical quadratic utility functions. Does the PIH hold (meaning: is $E_1(C_2) = C_1$)? (Again, if you can explain why or why not, that is sufficient without solving.)

2. (20%) An econometrician finds a relation

$$\Delta \log C_{it} - \Delta \log C_t \;=\; 0.4\,(\Delta \log Y_{it} - \Delta \log Y_t)\ ,$$

where C_{it} is the consumption of individual i and Y_{it} is the income of individual i and C_t is aggregate consumption. (Assume aggregate income growth is not equal to individual income growth; in other words: the right-hand side side is not 0. We also assume that income is exogenous.)
Assuming the coefficient 0.4 is statistically significant what does this results imply about the validity of

a) the Permanent Income Hypothesis?
b) Perfect Risk Sharing (under the standard assumption that all agents have identical CRRA utility functions)?

3. (20%). Assume that there are two states of the economy next year, "good" and "bad," each with probability 0.5. In the good state aggregate consumption grows 4% and in the bad state it grows 0%. Now consider assets D and E. For these we know the payouts. For D the payout is 5 in the bad state and 15 in the good state, while for E the payout is 5 in the bad state and 5 in the good state. Use the CCAPM as it was derived in the handout. The safe rate of return is 1%.
a) What would be the prices of assets D and E?
b) What would be the returns (you can give gross or net, but state which) of assets D and E?

4. (20%) Assume that an agent's wage income follows the AR(1) process

$$y_t = 300 + 0.5y_{t-1} + e_t \quad (*)$$

where e_t is white noise with variance 3.

Assume the agent's wage was 100$ period 0.
a) What is the agents expected wages in period t (for any $t > 0$)?
b) If the rate of interest is 10 percent and the PIH holds, what is the agent's level of consumption in period 0 assuming that his or her assets at the beginning of period 0 was 1000$.

5. (10%) Consider the CAPM-model. Assume the safe rate of interest is 10%, the mean return to the market portfolio is 20% and the variance of the return to the market portfolio is 0.02. Now consider assets D and E. For these we know the distribution of the pay-outs. For D the payout is normally distributed with mean 100 and variance 10, while for E the payout is normally distributed with mean 200 and variance 40. Assume the covariance of the payout to asset D with the market return is 1 while the covariance of payout to asset E with the market return is 2.
What would be the prices of assets D and E?