Final Exam, May 1 — 5 questions. All sub-questions carry equal weight unless otherwise indicated.

1. (20%) Consider an agent with income ("output" in Obstfeld-Rogoff) $Y_1 = 10$, $Y_2^A = 25$, and $Y_2^B = 10$, where A and B are states of the world with probabilities $\pi^A = 0.4$ and $\pi^B = 0.6$. Assume $p^A = p^B$, $r = 10\%$ and the discount rate is $\beta = \frac{1}{1+r}$. Assume that the agent has utility function $U(C) = \log C$ so that the agent maximizes $\log C_1 + \beta E_0 \log C_2$.

a) Find C_1, C_2^A, and C_2^B.
b) Explain intuitively why the agent has positive or negative (you need to explain which) borrowing in period 1?
c) Now assume $\frac{p^A}{p^B} = \frac{2}{3}$. Now find C_1 and C_2^S for $S = A, B$.
d) Explain intuitively why C_1, C_2^A, and C_2^B now are higher/lower/the came as you found in question a.

2. (20%) Derive the formula for how consumption changes as function of innovations to income when the PIH holds.

3. (10%) Assume that income follows the ARMA process

$$y_t = 0.3\, y_{t-1} + 0.3\, y_{t-2} + e_t$$

where e_t is white noise with variance 2.

a) If the time series y_t; $t = ..., -1, 0, 1, ...$ is stationary, what is the mean and the variance of y_0?
b) Now assume that y_0 and y_{-1} are given outcomes. What is the mean and the variance of y_1? And of y_2? (I am asking about the conditional means and variances as functions of the initial values.)

PLEASE TURN OVER

4. (20%) An economy with no storage consists of 2 identical consumers who live for 2 periods and earn \$30 in period 1 and in period 2 they make \$10 with probability 0.2 (state A), and \$100 with probability 0.8 (state B). Each consumer maximizes

$$U(C_1) + \frac{1}{1.1} E_1 U(C_2)$$

where

$$U(C) = \log C .$$

a) Assume that agents have access to only one asset. The asset pays out \$100 in states B) and nothing in state A. What is the expected net rate of return on that asset?
b) Now assume that the agents have access to a full set of Arrow securities, what would then be the expected rate of the return on the asset in part a)?

5. (15%) Assume that 2 agents (or countries) live for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. Assume that the endowment of the first agent is $y_0 = 4$, $y_1 = 6$ in state A and $y_1 = 2$ in state B. The endowment of the second agent in period 0 is $y_0^* = 4$ and in period 1 his or her endowment is $y_1^* = 4$ in both state "A" and state "B." Assume that state A happens with probability 0.5.
Assume each agent maximizes a utility function

$$U(C_0) + E_0 U(C_1) .$$

where

$$U(C) = C - \frac{1}{20} C^2 .$$

a) Find the rate of interest.
b) Explain intuitively why the interest rate is negative or positive.
Now assume that the agents only have access to safe bonds.
c) Explain intuitively why the interest rate will be higher or lower than in the case of perfect markets.

(15%) Explain the test of Barbara Mace in JPE 1991. Next explain (you can use words only for this) how Sala-i-Martin and Sachs studied the role of the U.S. federal government in smoothing the disposable income of U.S. states. Finally explain how Asdrubali, Sørensen, and Yosha (1996) examined channels or risk sharing and how they nested those two other papers (ignore the fact that Mace looked individuals rather than states).