ECONOMICS 7344, Spring 2007
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Final Exam, May 3 — 4 questions. All sub-questions carry equal weight.

1. (15%) Assume that income follows the AR(2) process

$$y_t = 3 + 0.5y_{t-1} + 0.1y_{t-2} + e_t \quad (*)$$

where e_t is white noise with variance 3.

i) Is the model stable (you need to show why or why not)?
ii) Assuming the model is stationary, what is the mean of y?
iii) What is conditional variance of y_2 if you condition on $y_0, y_{-1}, ...$
iv) What is $E_1 y_3$ if $y_1 = 4$ and $y_0 = 2$? (Note, the time indices are 0, 1, and 3).
v) Assuming the model is stationary, find the variance of y_t.

2. (11%) Derive the relation that show how the risk premium (the expected return minus the safe rate) of an asset depends on risk aversion. (I have in mind the approximate relation derived for the equity risk premium puzzle.)

3. (18%) A consumer lives for 3 periods and maximizes

$$C_1 - 1/20\, C_1^2 - \frac{1}{10}\, C_2^{-1} \; + \; \log(C_3) \; .$$

i) Assume the consumer optimally choose $C_1 = 20$ and $C_2 = 25$. What is the safe rate of interest from period 1 to period 2?
ii) If the rate of interest from period 1 to period 2 is 10% then what is C_3?
PLEASE TURN OVER

4. (56%) Consider the case of a 2 agents ("Home" and "Foreign"), 2 periods, 3 states-of-the-world model where agents can trade using a full set of Arrow securities. Assume that both agents have quadratic utility functions $U(C_0) + E_0U(C_1)$, where $U(C_t) = C_t - \frac{1}{200}C_t^2$.
Assume that the endowment of the first agent is $y_0 = 3$ and that the endowment of the second agent in period 0 is $y_0^* = 3$

The following table gives the possible endowments and the probabilities for Home and Foreign:

	Home			Foreign		
State of the world:	A	B	C	A	B	C
period 1 endowment	2	7	4	4	7	2
probability:	.25	.5	.25	.25	.5	.25

a) Find the prices of the Arrow-Debreu assets for each of the 3 states of the world.
b) Find the rate of interest. Explain in economic terms why it is positive or negative.
c) Does the perfect risk sharing relation $C_t^s = kY_t^{Ws}$ hold?
d) Assume that now only bonds can be traded. Find the rate of interest?
e) Explain in economic terms why the rate of interest would change or not change?
f) Assume that now there again are Arrow-Debreu securities but $U(C) = \log(C)$. Now find the prices of the Arrow-Debreu securities.
g) Explain in terms of economic intuition why the prices of states A, B, and C, now are higher or lower.
h) Find the consumption of home and foreign in period 1 and period 2 (all states of the world).