ECONOMICS 7395, Spring 2005
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Homework 3. February16—due February 23. (From Lars Hansen)

1. Assume
$$E[W_t - V_t\beta] = 0 \ ,$$
where W_t and V_t are stationary and ergodic $m \times 1$ and $m \times k$ matrices.
a) Show that for any $k \times m$ matrix A_0 such that $A_0 V_t$ is non-singular, the estimator β_T that satisfies
$$A_0 \frac{1}{T} \Sigma_{t=1}^{T} (W_t - V_t \beta_T) \ = \ 0 \ ,$$
converges almost surely to β_0.

Suppose A_T converges almost surely to A_0.
b) Show that the estimator β_T^* that satisfies
$$A_T \frac{1}{T} \Sigma_{t=1}^{T} (W_t - V_t \beta_T^*) \ = \ 0 \ ,$$
converges almost surely to β_0.

2. Consider the following single equation econometric model
$$y_t \ = \ X_t' \beta_0 \ + \ u_t \ ,$$
where X and u have finite second moments. In addition, there is an $m-$dimensional vector of random variables Z_t such that $E(Z_t u_t) = 0$ and $E(Z_t Z_t')$ is non-singular. Assume that $E(Z_t X_t)$ has rank k, where k is the number of elements in β_0. Show that the two-stage least stage estimator of β_0 is a special case of the estimator given in part b) of problem 1 where $W_t = Z_t y_t$ and $V_t = Z_t X_t'$. Characterize A_T for the two-stage least squares estimator and show that $A_0 = E(XZ')E(ZZ')^{-1}$.