

HOMEWORK 4. Wednesday February 17, due Wednesday February 24.

1. (15% of the January 2015 Core exam. No-one got parts B and C right, but I hope you will.) A consumer lives for 3 periods (periods 1, 2, and 3), earns \$100 in the first period and the distribution of future earnings follows a uniform distribution on the interval [90,110] in periods 2 and 3. The consumer has a quadratic utility function and is—in period 1—allowed to freely borrow and lend at an interest rate that equals his or her rate of time preference which we for simplicity set to 0 (i.e., the net rate of interest is 0). The consumer is allowed to save in period 2 but not to borrow and the consumer has access to no other assets. Let  $C_i$  be the consumption of the representative consumer in period  $i$ .

A) Is  $C_1 = E(C_2)$ ?

B) Is  $E_0(C_2) = E_0(C_3)$ ? (Explain.)

C) Does the consumer save in period 1? (Argue why, and if savings are not zero, explain if they are positive or negative.)

2. (20% of 2010 Final) Assume that an agent's wage income follows the AR(1) process

$$y_t = 300 + 0.5y_{t-1} + e_t \quad (*)$$

where  $e_t$  is white noise with variance 3.

Assume the agent's wage was 100\$ period 0.

a) What is the agent's expected wages in period  $t$  (for any  $t > 0$ )?

b) If the rate of interest is 10 percent and the PIH holds, what is the agent's level of consumption in period 0 assuming that his or her assets at the beginning of period 0 was 1000\$.

3. (15% of the January 2014 core exam) Assume that a representative agent's income follows a stationary AR(1) model with mean 0 and assume the PIH holds. Further assume that the agent's consumption satisfies

$$\Delta c_t = 0.2y_t - 0.12y_{t-1} .$$

a) What is coefficients in the AR(1) model for income?

b) What is the rate of interest?

4. (13% of the January 2015 exam) Assume that income follows the ARMA(1,1) process

$$y_t = 3 + \frac{1}{3}y_{t-1} + e_t + e_{t-1} \quad (*)$$

where  $e_t$  is white noise and  $y_{-1} = 1, e_{-1} = 0$ , and  $e_0 = 3$ .

Also assume that the rate of interest is  $\frac{1}{3}$  (i.e., the net interest rate is 33.333 percent) and equal to the discount rate.

Further assuming that a given agent has quadratic preferences and can freely lend and borrow at the fixed interest rate. Assume the agents initial assets (in period  $-1$ ) are 1000 dollars.

A) (6%) What is the change in consumption from period  $-1$  to period 0?

B) (7%) Assume the agents assets at the start of period 0 are 1000 dollars. What is the level of consumption in period 0?