

ECONOMICS 7344 – MACROECONOMIC THEORY II, Spring 2015

Homework 1. Wednesday January 21. Due Wednesday January 28.

1. Define the lag polynomial $a(L) = a_0 + a_1 L$ and $b(L) = b_0 + b_1 L + b_2 L^2$. (Notice: in the notes, and in class, it is often assumed $a_0 = 1$ and $b_0 = 1$. This is just for simplification and doesn't matter for any results since you can always re-scale the data and the lag-polynomial such that the first coefficient becomes unity (write $a(L)$ as $a_0 a'(L)$ where the lag polynomial $a'(L) = 1 + \frac{a_1}{a_0} L$ and similarly for $b(L)$). The constant a_0 will not affect the properties of the lag-polynomial that we care about. Also notice: The coefficients are real numbers (occasionally complex numbers) and can be negative or positive, it is therefore purely a matter of taste if you write $a(L) = a_0 + a_1 L$ or $a(L) = a_0 - a_1 L$.)

Assume $a_0 = 4$, $a_1 = -1$, $b_0 = 1$, $b_1 = -7$, and $b_2 = 1$.

- i) If $x_{t-1} = 2$, $x_{t-2} = -3$, $x_{t-3} = -2$, and $x_{t-4} = 9$, what is $a(L)x_t$? and $b(L)x_t$?
- ii) What is $c(L) = a(L)b(L)$? You *have to* do that by finding $a(L)b(L)x_t$ [for general x_t not the specific realizations given] using the definition that $a(L)b(L)x_t = a(L)[b(L)x_t]$ and simplifying).

Define $a(x) = a_0 + a_1 x$ and $b(x) = b_0 + b_1 x + b_2 x^2$.

- iii) Find $a(x)b(x)$ and compare the coefficients with $a(L)b(L)$.

- iv) Find the roots of $c(L)$. Is the AR-model $c(L)x_t = 8 + u_t$ stable?

2. Define the polynomials $a(x) = 1 + .2x$ and $b(x) = 1 - .5x - .5x^2$ and find the roots [meaning the solution(s) to, say, $a(x) = 0$] in each polynomial. What are the roots of the polynomial $c(x) = a(x) * b(x)$?