ECONOMICS 7344, Spring 2014
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HOMEWORK 8. Due March 26.

1. If a two-period (from period 0 to period 2) discount bond has a price of 100 and a payout of 122 and a one period discount bond has a price of 100 and a payout of 110. What is the (implied) forward rate of interest from period 1 to period 2?

2. (20% of midterm 2, 2013) Assume a representative agent lives of three periods (period 0, 1, and 2) and that he or she can trade in 1- or 2-period bonds.
Assume that the agent maximizes

$$U(C_0) + \frac{1}{1.1} E_0 U(C_1) + \frac{1}{(1.1)^2} E_0 U(C_2) \ ,$$

where $U(C) = \log C$.
Assume $C_0 = 10$, C_1 is 9 with probability 0.5 and 11 with probability 0.5, while C_2 is independent of C_1 and equals 5 with probability 0.5 and 15 with probability 0.5.
i) What is the safe rate of interest from period 0 to period 1?
ii) What is the forward rate of interest from period 1 to period 2?

3. (32% of midterm 2, 2010) Consider the CAPM-model.
Assume the world only have two outcomes ("states of the world"). Let X be an asset whose payout PO_X is 100 if "shine," a situation where the market return is 10 percent. "Shine" has probability 0.5. If "rain," the pay-out to the asset is 50 and the market return is 0; "rain" also has probability 0.5. Assume that the safe rate of interest is 1 percent.

a) What is the expected return ($\mathrm{E}R_X$) to an investment in X?

b) What are the possible returns R_X and their probabilities (in other words, what is the distribution of R_X)?

c) What is the price of asset X in the initial period?

Now assume that Y is an asset whose payout PO_Y is 200 if "shine" and 100 if "rain."

d) What is the expected return $\mathrm{E}R_Y$? (Hint: you can provide the answer easily, if you consider the relation between the payout to X and the payout to Y.)

4. (7% of second core exam 2003.) Asset A and asset B exist for one period and their

returns have identical covariances with the market return. The rate return of asset B has a variance that is twice as large as the variance of the rate of return of asset A. Which asset will—if the CAPM holds—have the highest expected rate of return?

5. Assume that the return on the market (R_M) is 10% and that a safe asset exists with a return of 6%. Assume that the standard CAPM is true.
a) Let X is an asset whose payout is determined by you flipping a coin and paying 1$ each heads and nothing if tails. What is the return (R_X) to an investment in X?
b) Now let the return (R_i) to an asset be $R_i = .5\,R_M + .5\,R_X$. What is the expected value $E(R_i)$.
c) If the asset X now paid out 100$, rather than just 1$, in the case of heads, and still nothing in the case of tails. What would now be the answer to b)?