

HOMEWORK 7. Due Wednesday March 19.

1. (20% of midterm 2, 2009) Consider the Consumption CAPM-model. a) Assume the world only have two outcomes (“states of the world”). Let X be an asset whose payout PO_X is 100 if “shine” a situation where consumption growth is 4 percent. “Shine” has probability 0.5. If “rain” the pay-out to the asset is 200 and consumption growth is 0, “rain” also has probability 0.5. Assume that the safe rate of interest is 1 percent.

a) What is the expected return (ER_X) to an investment in X ?

b) What are the possible returns R_X and their probabilities (in other words, what is the distribution of R_X).

2. A consumer lives for 2 periods and earns $Y_1 = 9\$$, in period 1, and in period 2 he or she earns $Y_2^a = 5\$$ with probability $3/5$ (state a) and $Y_2 = 15\$$ with probability $2/5$ (state b). The consumer starts with 0 assets and maximizes

$$U(C_1) + \frac{1}{1.05} E_1 U(C_2) ,$$

where

$$U(C) = 100C - \frac{1}{2}C^2 .$$

Assume that the safe rate of interest is 5 percent.

a. Find consumption C_1 in period 1 and the consumption plan for period 2 (i.e., the consumption if states a and b).

b. Now assume that the rate of interest is 10 percent. Find consumption in period 1 and the consumption plan for period 2.

c. Explain (in words) why consumption goes up or down when the interest rate increases.

d. Starting from the original assumption of question a., explain whether (and why!) consumption will go up or down following an increase in the interest rate.

Now assume some new asset is introduced. Assume again a discount rate equal to the interest rate of 5 percent. Also assume that the consumption that you found in question a. is unchanged. (Note: a new asset may change the budget constraint and therefore consumption, but we ignore that here). If the asset has a payout of 10 in state a and 20 in state b .

- e. Find the period 1 price of the asset.
- f. Assuming that consumption is the same as before (even if that is not likely to be correct) find the price of the asset under the assumption that $U(C) = \log(C)$.
- g. Explain why the price now is higher or lower?
- h. If the asset instead had a payout of 20 in state a and 10 in state b the price would change. Explain (assuming logarithmic utility) if the price would go up or down and give *two* reasons for why this would happen. (If you feel like it, you can solve for the price, but if you are confident with the material taught you can just explain what would happen and why).
3. (15% of 2013 midterm 2) Assume that inflation in period t is I_t and I_t is positively correlated with the (gross) market return R_t^M . Inflation has a positive mean (i.e., different from 0). Set the safe rate of interest to 0 for simplicity. A researcher may find the following patterns (with significant positive coefficients) for assets i :

$$\begin{aligned}
 (A) \quad R_t^i &= \alpha^i I_t + u_{it} , \\
 (B) \quad R_t^i &= \beta^i R_t^M + u_{it} , \\
 (C) \quad R_t^i &= \alpha^i I_t + \beta^i R_t^M + u_{it} ,
 \end{aligned}$$

where the u error terms are white noise.

- i) In case A, does the CAPM hold? does the CAPM not hold? or, we cannot tell?
- ii) In case B, does the CAPM hold? does the CAPM not hold? or, we cannot tell?
- iii) In case C, does the CAPM hold? does the CAPM not hold? or, we cannot tell?