

HOMEWORK 4. Wednesday February 5, due February 12.

1. A consumer lives for 3 periods and earns 200\$, X \$, and 400\$ in period 1, 2, and 3 respectively. X is a Normally distributed random variable with mean 300 and variance 2. The consumer has a quadratic utility function and is—in period 1—allowed to freely borrow and lend at an interest rate that equals his or her rate of time preference (to be paid back in period 2). The consumer is not allowed to borrow or lend in period 2. Let C_i be the consumption of the representative consumer in period i . Is $C_1 = E(C_2)$ and is $C_2 = E(C_3)$? Find C_1 , C_2 , and C_3 (if C_i is a random variable, describe its distribution).

[NOTE: This question is modified from one on the 2004 make-up core exam.]

2. Consider a consumer who maximize the *expected* utility of present and future consumption using the criterion function is

$$U(c_t) + \frac{1}{1+\rho} EU(c_{t+1}) + \left(\frac{1}{1+\rho}\right)^2 EU(c_{t+2}) + \dots$$

Assume that the consumer can choose consumption in periods t , $t+1$, and $t+2$ without any constraints. Now assume that the consumer considers an investment in period t that will pay back in period $t+2$ with a (stochastic) interest rate of $r_{t,t+2}$.

- i) Find the corresponding Euler equation linking consumption in period t and in period $t+2$.
- ii) Show that the Euler equations for period t (relative to period $t+1$) and for period $t+1$ (relative to period $t+2$) are consistent with what you found in part i). (Hint: You need to express the rate of return from period $t+1$ to $t+2$ as a function of the other rates of return.)

3. (20% of last years midterm 1) A consumer lives for 2 periods and earns $Y_1 = 10$ \$, in period 1, and in period 2 he or she earns $Y_2^a = 10$ \$ with probability 1/2 (state a) and $Y_2^b = 30$ \$ with probability 1/2 (state b). The consumer starts with 0 assets and maximizes

$$U(C_1) + E_1 U(C_2) ,$$

where

$$U(C) = \log(C) .$$

Assume that the safe rate of interest is 10 percent.

a.) Let B denote the amount lent in period 1 (or, equivalently, the amount of a safe bond bought). Assuming that the agent doesn't have access to any other assets, find B and the consumption in

each period (for period 2, that means the consumption plan listing consumption in state a and state b .)

b.) Now assume that a stock (equity) exists besides the safe bond. Let the amount of equity bought be S (it can be negative). Assume that the stock has a (net) rate of return of 10% if state a occurs [meaning that agent gets back the principal] and 20% if state b occurs. Find B and S (and the implied consumption plan).

Derive (but don't solve because it gets messy) two equations in two unknowns that determines B , and S .