

ECONOMICS 7344 – MACROECONOMIC THEORY II, Spring 2014

Homework 2. Thursday January 23. Due Wednesday January 29.

1. (24% of Midterm 1, Spring 2005 (numbers slightly changed)) Assume that income follows the AR(1) process

$$y_t = 2 + 0.8y_{t-1} + e_t \quad (*)$$

where  $e_t$  is white noise with variance 3.

- Is this time-series process stable?
- Assume that  $y_0$  is a random variable. For what values of the mean  $E(y_0)$  and the variance  $\text{var}(y_0)$  will the time series  $y_t$ ;  $t = 0, 1, 2, \dots$  be stationary?
- What is  $E_1 y_3$  if  $y_1 = 4$  and  $y_0 = 2$ ?
- Write the infinite Moving Average model that is equivalent to the AR(1) model (\*) [assuming that the process now is defined for any integer value of  $t$ ]. (Half the points are from getting the correct mean term.)

2. (30% of Midterm 1, 2010, the percentages are from the exam) Assume that  $y_t$  follows the AR(2) process

$$y_t = 200 + 0.5y_{t-1} + 0.3y_{t-2} + e_t \quad (*)$$

where  $e_t$  is white noise with variance 4.

- (8%) Is this process stable? (You need to show why).
- (8%) Find the mean and variance of  $y_t$  assuming that  $y_t$  is stationary.

Assume that you are told that  $y_1 = 500$  and  $y_0 = 400$ . (But for question d) you should still think of these as realizations from the stationary distribution.)

- (6%) What is the conditional expectation  $E(y_3|y_0, y_1)$ ?

*The one below is for extra points. I don't think anybody could do during the midterm,*

*you have to use the formula for conditional expectation in a bivariate normal, if you have not been taught that, you can skip it.*

d) (8%) Prove that  $E(y_1|y_0) = E(y_0|y_1)$ ? (Hint: One way is to make the assumption that  $y_1$  and  $y_0$  are jointly normally distributed and drawn from the stationary distribution and use what you know from 6331. Same issue for question 4 d.)

3. (15% of the January 2014 core exam) Assume  $y_t$  and  $x_t$  are independent stationary moving average processes, which follow the models

$$x_t = 10 + u_t + u_{t-1} ,$$

and

$$y_t = 15 + v_t + v_{t-1} ,$$

where the innovations  $u_t$  and  $v_t$  have variance 1.

a) What time series model does  $z_t = x_t + y_t$  follow? (You need to substantiate your claim to get more than a single point.)

Now assume that  $x_t$  and  $y_t$  follow AR(1) models:

$$x_t = 0.4 x_{t-1} + u_t$$

and

$$y_t = 0.4 y_{t-1} + v_t$$

b) Is  $z_t = x_t + y_t$  an AR(1) model? (You need to substantiate your claim to get more than a single point. You may assume that the variables are normally distributed.)

Now assume that  $x_t$  and  $y_t$  follow AR(1) models:

$$x_t = 0.4 x_{t-1} + u_t$$

and

$$y_t = 0.2 y_{t-1} + v_t$$

c) Is  $z_t = x_t + y_t$  an AR(1) model? (You need to substantiate your claim to get more than a single point.)

4. (20% of midterm 1, 2008) Assume that  $y_t$  follows the AR(2) process

$$y_t = 200 + 0.5y_{t-1} + 0.1y_{t-2} + e_t \quad (*)$$

where  $e_t$  is white noise with variance 2.

a) (8%) Find the mean and variance of  $y_t$  assuming that  $y_t$  is stationary.

Now assume that you are told that  $y_1 = 200$  and  $y_0 = 200$ .

b) (4%) What is the conditional expectation  $E(y_2|y_0, y_1)$ ?

c) (4%) What is the conditional expectation  $E(y_3|y_0, y_1)$ ?

d) (4%) What is the conditional expectation  $E(y_1|y_0)$ ? (Hint: Make the assumption that  $y_1$  and  $y_0$  are jointly normally distributed and use what you know from 6331.)