ECONOMICS 7344, Spring 2014
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HOMEWORK 11. Due Wednesday April 23.

1. Consider the case of an economy with four states-of-the-world. Assume that an asset S_1 exists that pays 2 units in period 1 if state A occurs, 1 unit if state B occurs, and nothing if state C or D occurs. Another asset S_2 exists which pays 1 unit in period 1, if state C occurs, and nothing in states A, B, and D. A third asset S_3 pays 0 units in period 1 if state C occurs, and 2 units in states A, B, and D. Finally, a discount bond paying one unit in period 1 for sure can be traded.

a) Is the set of assets equivalent to a full set of Arrow securities?

b) Now assume that asset S_3 instead pays 1 unit in period 1, if state A occurs, and 0 units in states B, C, and D. Are the markets perfect (equivalent to a full set of Arrow securities) in this case?

2. (20% of final, 2010) An econometrician finds a relation

$$\Delta \log C_{it} - \Delta \log C_t \; = \; 0.4 \, \Delta Y_{it-1} \; ,$$

where C_{it} is the consumption of individual i and Y_{it} is the income of individual i and C_t is aggregate consumption. (Assume aggregate consumption growth is not equal to individual consumption growth; in other words: the left-hand side side is not 0.)
Assuming the coefficient 0.4 is statistically significant what does this results imply about the validity of
a) the Permanent Income Hypothesis?
b) Perfect Risk Sharing (under the standard assumption that all agents have identical CRRA utility functions)?

3. (60% of final 2008) Consider the case of a 3 agents ("Home," "Foreign," and "Really Foreign"), 2 periods, 2 states-of-the-world model where agents can trade using a full set of Arrow securities. Assume that all agents have quadratic utility functions $U(C_0)+\beta E_0 U(C_1)$, where $U(C_t) = C_t - \frac{1}{200} C_t^2$ and $\beta = \frac{1}{1.1}$.
Assume that the endowment of the first agent is $y_0 = 3$, that of the second agent in period 0 is $y_0^* = 3$, and that of the third agent $y^{**} = 6$.

The following table gives the possible endowments and the probabilities for Home, Foreign and Really Foreign:

	Home		Foreign		Really Foreign	
State of the world:	A	B	A	B	A	B
period 1 endowment	2	7	7	2	9	9
probability:	.5	.5	.5	.5	.5	.5

a) Find the prices of the Arrow-Debreu assets for each of the 2 states of the world.
b) Find the rate of interest.
c) Argue in economic terms why the interest rate is larger or smaller than 0 and larger or smaller than the discount rate.
d) Assume that now only bonds can be traded. Find the rate of interest?
e) Find the consumption in period 1 and period 2 of the Home agent. (If you write down one equation in one unknown, that is considered a full answer, don't spend time on solving.)
f) Assume that now there again are Arrow-Debreu securities but $U(C) = \log(C)$. Find the prices of the Arrow-Debreu securities.
g) Find the rate of interest.
h) Find the consumption of all agents in all periods and all states of the world.
i) Assume that the agents only have access to a bond. State 3 equations in 3 unknowns that would determine the consumption of the agents and the interest rate. (The equations are messy to solve, so do not solve them.)
j) Assume now that agents have access to an Arrow-Debreu security that pays out one unit in state A and the agents also have access to a bond. Find the consumption of all agents in all states of the world. (Hint: If you think carefully about this, you may not have to do a lot of calculations.)