

HOMEWORK 9. Due Monday April 17.

1. Assume that two agents live for two periods in an economy with perfect Arrow-Debreu markets and no storage. Assume that the endowment of the first agent is $y_0 = 1, y_1 = 6$ and that the endowment of the second agent in period 0 is $y_0^* = 2$ and in period 1 his or her endowment is $y_1^* = 10$ in the “good state” g . In the “bad state” b the endowment of the second agent is $y_1^* = 0$. Assume that the good state happens with probability $1/2$.

Assume each agent maximizes a utility function

$$-C_0^{-1} + E_0\{-C_1^{-1}\} .$$

- i) Find the period 0 prices of the Arrow securities that pays out one unit in the good and the bad state, respectively. (I suggest that you follow Obstfeld-Rogoff and parameterize such that the period 0 price of 1 unit delivered in the good state is $p^g/(1+r)$ and in the bad state it is $p^b/(1+r)$ which implies that $p^g + p^b = 1$ when r is the safe rate of interest. The price of a unit of period 0 consumption is normalized to 1.)
- ii) Find the safe rate of interest r .
- iii) Find the value (in terms of period 0 output) of the second (“*”) agent’s output.
- iv) Find the level of consumption of each of the agents in periods 0 and 1 and both states of the world.
- v) Argue, using words, whether the consumption of agent 1 would increase or decrease (compared to the model above) if the utility function were $-\frac{1}{2}C_0^{-2} - E_0\frac{1}{2}C_1^{-2}$. (Try to spell out the main economic intuition.)

Now assume the original utility function again.

- vi) Demonstrate what would happen to the interest rate (i.e., would it go up or would it go down) if world output in period 1 were constant rather than a random variable—assume that the mean value of period 1 output is the same. (If you have trouble with this, you will get points if you argue coherently in intuitive terms what would happen.)
- vii) What would happen to the interest rate if world output in period 0 increased (with no change in period 1). (As before, a mathematical demonstration is perfect, but words can get you most of the points if they are precise.)
- viii) What would happen to the interest rate (going up or down) if the agents discounted second period consumption with positive discount rate instead of with the 0 discount rate used so far? (As before, a mathematical demonstration is perfect, but words can get you most of the points if they are precise.)
- ix) Now assume that no Arrow securities exists, but that the agents in period 0 can trade a safe

asset. Find the safe interest rate in this case (define a bond B which pays out 1 unit for sure in period 1 and trades at a price $1/(1+r)$). Use the utility function from v).

[Note, the first order conditions will give you some non-linear equations, that aren't easy to solve by hand. You will get full points for stating the restrictions and first order conditions for the problem.]

x) What is the period 1 consumption of agent 2 in this situation (in each state of the world)? [If you don't find B in ix) set use $B = .1$]