

HOMEWORK 8.

1. Consider an agent with income (“output” in Obstfeld-Rogoff) $Y_1 = 10$, $Y_2^A = 20$, and $Y_2^B = 0$, where A and B are states of the world with $\pi^A = 0.4$ and $\pi^B = 0.6$. Assume $p^A = p^B$, $r = 10\%$ and the discount rate is $\beta = \frac{1}{1+r}$.

a) Assume the agent has quadratic utility. Does the “PIH-relation” $C_1 = EC_2$ hold?

b) Find C_2^A/C_2^B .

c) How many units of each Arrow-security does the agent purchase and how many units of the period 1 good? (this can be a negative number so “purchase” may mean sell.)

2. Now assume that the agent has utility function $U(C) = -\frac{1}{3}C^{-3}$.

a) Does the “PIH-relation” $C_1 = EC_2$ hold?

b) What is the intuition for the answer you gave in part a)?

c) Assume $\frac{p^A}{p^B} = \frac{2}{3}$. Now find C_1 and C_2^S for $S = A, B$ and check if $C_1 = EC_2$.

3. (30% of Final 2011) Consider the case of a 2 agents (“Home” and “Foreign”), 2 periods, 3 states-of-the-world model where agents can trade using a full set of Arrow securities. Assume that both agents have exponential utility functions $U(C_0) + E_0U(C_1)$, where $U(C_t) = -\exp(-C_t)$.

Assume that the endowment of the first agent is $y_0 = 3$ and that the endowment of the second agent in period 0 is $y_0^* = 3$

The following table gives the possible endowments and the probabilities for Home and Foreign:

	Home			Foreign		
State of the world:	A	B	C	A	B	C
period 1 endowment	2	7	4	4	7	2
probability:	.25	.5	.25	.25	.5	.25

- a) Find the prices of the Arrow-Debreu assets for each of the 3 states of the world.
- b) Find the rate of interest. Explain in economic terms why it is positive or negative.
- c) Assume that now only bonds can be traded. Find the rate of interest.
- d) Explain in economic terms why the rate of interest would change or not change?

3. (Part of question 3, Final 2004) Consider the case of the 2 agents, 2 periods, 2 states-of-the-world model of Obstfeld-Rogoff Chapter 5.2 (where agents can trade using a full set of Arrow securities). Assume that both agents have quadratic utility functions $U(C_0) + E_0U(C_1)$, where $U(C_t) = aC_t + bC_t^2$. Assume that $a = 10$ and $b = -0.5$.

Assume that the endowment of the first agent is $y_0 = 2, y_1 = 2$ and that the endowment of the second agent in period 0 is $y_0^* = 2$ and in period 1 his or her endowment is $y_1^* = 4$ in the “good state” g . In the “bad state” b the endowment of the second agent is $y_1^* = 4/3$. Assume that the good state happens with probability 0.25.

- a) Find the price of the Arrow securities for state g and state b .
- b) Explain intuitively why the price of the Arrow security for one of the states may be higher than for the other state. You need to give two reasons and explain the logic.
- c) Find the safe rate of interest.