

HOMEWORK 9. Due Monday April 9.

1. Assume that two agents live for two periods in an economy with perfect Arrow-Debreu markets and no storage. Assume that the endowment of the first agent is $y_0 = 1, y_1 = 6$ and that the endowment of the second agent in period 0 is $y_0^* = 2$ and in period 1 his or her endowment is $y_1^* = 10$ in the “good state” g . In the “bad state” b the endowment of the second agent is $y_1^* = 0$. Assume that the good state happens with probability $1/2$.

Assume each agent maximizes a utility function

$$-C_0^{-1} + E_0\{-C_1^{-1}\}.$$

- i) Find the period 0 prices of the Arrow securities that pays out one unit in the good and the bad state, respectively. (I suggest that you follow Obstfeld-Rogoff and parameterize such that the period 0 price of 1 unit delivered in the good state is $p^g/(1+r)$ and in the bad state it is $p^b/(1+r)$ which implies that $p^g + p^b = 1$ when r is the safe rate of interest. The price of a unit of period 0 consumption is normalized to 1.)
- ii) Find the safe rate of interest r .
- iii) Find the value (in terms of period 0 output) of the second (“*”) agent’s output.
- iv) Find the level of consumption of each of the agents in periods 0 and 1 and both states of the world.
- v) Argue, using words, whether the consumption of agent 1 would increase or decrease (compared to the model above) if the utility function were $-\frac{1}{2}C_0^{-2} - E_0\frac{1}{2}C_1^{-2}$. (Try to spell out the main economic intuition.)
- vi) Demonstrate what would happen to the interest rate (i.e., would it go up or would it go down) if world output in period 1 were constant rather than a random variable—assume that the mean value of period 1 output is the same. (If you have trouble with this, you will get points if you argue coherently in intuitive terms what would happen.)
- vii) What would happen to the interest rate if world output in period 0 increased (with no change in period 1). (As before, a mathematical demonstration is perfect, but words can get you most of the points if they are precise.)
- viii) What would happen to the interest rate (going up or down) if the agents discounted second period consumption with positive discount rate instead of with the 0 discount rate used so far? (As before, a mathematical demonstration is perfect, but words can get you most of the points if they are precise.)