

HOMEWORK 7. Due Monday March 25.

1. **(5% of January 2010, core)** Consider the CAPM-model. Assume the safe rate of interest is 2%, the mean return to the market portfolio is 6% and the variance of the (percent) return to the market portfolio is 2. Now consider an asset with a payout which is normally distributed with mean  $X$  and variance 10 (again in percentage terms). Assume the covariance of the payout to asset  $S$  with the market return is 2.

a) If the price of the asset is 100 \$ what would be the value of  $X$ ?

b) If everything is the same as before except that the asset's payoff have a variance of 20. What would be the price of the asset be then?

2. **(20% of midterm 2, 2009)** Consider the Consumption CAPM-model. a) Assume the world only have two outcomes ("states of the world"). Let  $X$  be an asset whose payout  $PO_X$  is 100 if "shine" a situation where consumption growth is 4 percent. "Shine" has probability 0.5. If "rain" the pay-out to the asset is 200 and consumption growth is 0, "rain" also has probability 0.5. Assume that the safe rate of interest is 1 percent.

a) What is the expected return ( $ER_X$ ) to an investment in  $X$ ?

b) What are the possible returns  $R_X$  and their probabilities (in other words, what is the distribution of  $R_X$ ).

3. A consumer lives for 2 periods and earns  $Y_1 = 9\$$ , in period 1, and in period 2 he or she earns  $Y_2^a = 5\$$  with probability  $3/5$  (state  $a$ ) and  $Y_2 = 15\$$  with probability  $2/5$  (state  $b$ ). The consumer starts with 0 assets and maximizes

$$U(C_1) + \frac{1}{1.05} E_1 U(C_2) ,$$

where

$$U(C) = 100C - \frac{1}{2}C^2 .$$

Assume that the safe rate of interest is 5 percent.

a. Find consumption  $C_1$  in period 1 and the consumption plan for period 2 (i.e., the consumption if states  $a$  and  $b$ ).

b. Now assume that the rate of interest is 10 percent. Find consumption in period 1 and the consumption plan for period 2.

- c. Explain (in words) why consumption goes up or down when the interest rate increases.
- d. Starting from the original assumption of question a., explain whether (and why!) consumption will go up or down following an increase in the interest rate.

Now assume some new asset is introduced. Assume again a discount rate equal to the interest rate of 5 percent. Also assume that the consumption that you found in question a. is unchanged. (Note: a new asset may change the budget constraint and therefore consumption, but we ignore that here). If the asset has a payout of 10 in state  $a$  and 20 in state  $b$ .

- e. Find the period 1 price of the asset.
- f. Assuming that consumption is the same as before (even if that is not likely to be correct) find the price of the asset under the assumption that  $U(C) = \log(C)$ .
- g. Explain why the price now is higher or lower?
- h. If the asset instead had a payout of 20 in state  $a$  and 10 in state  $b$  the price would change. Explain (assuming logarithmic utility) if the price would go up or down and give *two* reasons for why this would happen. (If you feel like it, you can solve for the price, but if you are confident with the material taught you can just explain what would happen and why).