

HOMEWORK 5. Wednesday February 22, due February 27.

1. A consumer lives for 2 periods and earns $Y_1 = 15\$$, in period 1, and in period 2 he or she earns $Y_2 = 10\$$ with probability $1/2$ and $Y_2 = 20\$$ with probability $1/2$. The consumer maximizes

$$U(C_1) + \beta E_1 U(C_2) ,$$

where

$$U(C) = 100C - \frac{1}{2}C^2 .$$

a. Assume that the rate of interest is 0 and the discount factor is 1 and find C_1 and C_2 (for each of the period 2 outcomes).

b. What is the solution if the interest rate is 10 percent and the discount factor is still 1?

c. What is the solution if the discount rate is 10 percent and the interest rate is 10 percent?

d. And if the discount rate is 10 percent and the interest rate is 0?

e. What is your answer to part a if instead $Y_2 = 2\$$ with probability 0.5 and $Y_2 = 8\$$ with probability 0.5?

2. Repeat question 1 assuming log utility.

3. (40% of midterm 2, 2008) Assume that y_t follows the stationary AR(1) process

$$y_t = 200 + 0.2y_{t-1} + u_t$$

where u_t is white noise with variance 2.

a) (5%) Find the mean and variance of y_t .

Now assume that the PIH holds such that $\Delta C_t = \alpha u_t$. Assume the rate of interest is 10%.

b) (10%) Find the value of α (this should be a number).

c) (5%) What is the variance of consumption growth?

Now you are told that $y_2 = 210, y_1 = 200$ and $y_0 = 200$. (This holds for the remaining questions.)

d) (5%) What is ΔC_2 ?

e) (10%) Assume that the consumer has assets $A_2 = 0$ at the beginning of period 2. What is C_2 ?

f) (5%) What is the conditional expectation $E\{C_3|y_2, y_1, y_0\}$?

4. (20% of 2010, second core exam, January 2011) Assume that income follows the time-series process

$$y_t = 3 + 0.5y_{t-1} + e_t ,$$

where e_t is white noise with variance 4.

a) Assume that the PIH holds. Find the asymptotic value of the coefficient β in the (OLS) regression

$$c_t = \alpha + \beta y_t + u_t ,$$

where c_t is (PIH-) consumption and u_t an error term.

b) If x_t is white noise measurement error with variance 4, with distribution is independent of that of y_s (for all s) and we define $z_t = x_t + y_t$. Assume you estimate the OLS regression

$$c_t = \alpha + \gamma z_t + u_t .$$

What is the asymptotic value of γ ?