

HOMEWORK 4. Wednesday February 15, due February 20.

1. A consumer lives for 3 periods and earns 200\$,  $X$ \$, and 500\$ in period 1, 2, and 3 respectively.  $X$  is a Normally distributed random variable with mean 300 and variance 2. The consumer has a quadratic utility function and is—in period 1—allowed to freely borrow and lend at an interest rate that equals his or her rate of time preference (to be paid back in period 2). The consumer is not allowed to borrow or lend in period 2. Let  $C_i$  be the consumption of the representative consumer in period  $i$ . Is  $C_1 = E(C_2)$  and is  $C_2 = E(C_3)$ ? Find  $C_1$ ,  $C_2$ , and  $C_3$  (if  $C_i$  is a random variable, describe its distribution).

[NOTE: This question is modified from one on the 2004 make-up core exam. ]

2. Consider a consumer who maximize the *expected* utility of present and future consumption using the criterion function is

$$U(c_t) + \frac{1}{1 + \rho} EU(c_{t+1}) + \left(\frac{1}{1 + \rho}\right)^2 EU(c_{t+2}) + \dots$$

Assume that the consumer can choose consumption in periods  $t$ ,  $t + 1$ , and  $t + 2$  without any constraints. Now assume that the consumer considers an investment in period  $t$  that will pay back in period  $t + 2$  with a (stochastic) interest rate of  $r_{t,t+2}$ .

- i) Find the corresponding Euler equation linking consumption in period  $t$  and in period  $t + 2$ .
- ii) Show that the Euler equations for period  $t$  (relative to period  $t + 1$ ) and for period  $t + 1$  (relative to period  $t + 2$ ) are consistent with what you found in part i). (Hint: You need to express the rate of return from period  $t + 1$  to  $t + 2$  as a function of the other rates of return.)

3. Romer 8.10. (This homework builds on a very famous paper by Lucas and it is important that you try to understand the details—in other words, this will be assumed known for the exams.)