

HOMEWORK 10. Due Monday April 16.

1. This is a continuation of the homework question in the last home work.

Assume that 2 agents live for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. Assume that the endowment of the first agent is $y_0 = 1, y_1 = 4$ and that the endowment of the second agent in period 0 is $y_0^* = 2$ and in period 1 his or her endowment is $y_1^* = 6$ in the “good state” g . In the “bad state” b the endowment of the second agent is $y_1^* = 0$. Assume that the good state happens with probability $1/2$.

Assume each agent maximizes a utility function

$$-C_0^{-1} + E_0 - C_1^{-1} .$$

ix) Now assume that no Arrow securities exists, but that the agents in period 0 can trade a safe asset. Find the safe interest rate in this case (define a bond B which pays out 1 unit for sure in period 1 and trades at a price $1/(1+r)$). Use the utility function from v).

[Note, the first order conditions will give you some non-linear equations, that aren't easy to solve by hand. You will get full points for stating the restrictions and first order conditions for the problem.]

x) What is the period 1 consumption of agent 2 in this situation (in each state of the world)? [If you don't find B in ix) set use $B = .1$]

2. **38% of Core Exam, Summer 2007** Assume that 3 agents (or countries) live for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. Let superscripts denote the agents and subscripts indicate the time period. Assume that the endowment of the first agent is $y_0^1 = 3, y_1^1 = 5$ in state A and $y_1^1 = 1$ in state B. The endowment of the second agent in period 0 is $y_0^2 = 3$ and in period 1 his or her endowment is $y_1^2 = 4$ in state “A” and $y_1^2 = 2$ in state “B”. The endowment of the third agent in period 0 is $y_0^3 = 3$ and in period 1 his or her endowment is $y_1^3 = 2$ in state “A” and $y_1^3 = 4$ in state “B”. Assume that state A happens with probability $1/2$. Assume each agent maximizes a utility function

$$\log(C_0) + E_0 \log(C_1) .$$

- a) (18%) Find the consumption of all agents in both periods and in both states of the world.
- b) (10%) Which agent has the highest consumption? Explain the intuition for why. (Note, you only get points for the intuition in this question. If the intuition is correct, you will get full points even if the answer to part a) is wrong.)
- c) (10%) Find the rate of interest. Explain intuitively why it is negative or positive.

d) (10%) Now assume that there are no Arrow-Debreu assets in the economy, but that the agents can borrow and lend at the equilibrium interest rate (i.e., a “bond economy.”) State the first order conditions that determines the lending/borrowing of each of the agents and the equilibrium interest rate. (Do not try to solve the equations.)

3. (32% of January 2012 core) Consider the case of the 2 agents, 2 periods, 3 states-of-the-world. Assume that both agents have CRRA utility functions $U(C_0) + E_0U(C_1)$, where $U(C_t) = -\frac{1}{2}C_t^{-1}$. Assume that the endowment of the first agent is $y_0 = 3, y_1 = 3$ and that the endowment of the second agent in period 0 is $y_0^* = 3$ and in period 1 his or her endowment is $y_1^{A*} = 6$ in state A, $y_1^{B*} = 3$ in state B, $y_1^{C*} = 0$ in state C. Assume that each state happens with probability 1/3. Assume that an asset “S” exists that pays 2 units in period 1 if state A occurs, 1 unit if state B occurs, and nothing if state C occurs. Another asset “T” exists which pays 1 unit in period 1, if state C occurs, and nothing in states A and B. Finally, a discount bond paying one unit in period 1 for sure can be traded.

a) (8%) Find the prices of assets S and T .

b) (5%) What is the rate of interest?

c) (3%) Explain why the interest rate is negative or positive in part a).

d) (4%) Which agent has the higher consumption and why? (You do not have to find the consumption of the agents numerically if you can clearly explain it intuitively.)

e) (5%) If the agents instead have quadratic consumption functions (and the discount rate is still 0), what would be the rate of interest? (You do not have to solve the equations if you can see what the answer should be, but you need to explain clearly.)

f) (4%) Now assume that the agents *only* have access to a safe bond but assume the utility function of parts a)-d). Would the interest rate go up or down compared to the situation in part b)? Don't try to solve this, you have to provide the intuition.

e) (3%) Would any of the agents borrow from the other agent (i.e., sell bonds in period 0) and, if so, which agent? Don't try and solve this, but explain the intuition.