

HOMEWORK 5. Due Wednesday March 30. (This is a somewhat long homework.)

1. (40% of midterm 2, 2008) Assume that  $y_t$  follows the stationary AR(1) process

$$y_t = 200 + 0.2y_{t-1} + u_t$$

where  $u_t$  is white noise with variance 2.

a) (5%) Find the mean and variance of  $y_t$ .

Now assume that the PIH holds such that  $\Delta C_t = \alpha u_t$ . Assume the rate of interest is 10%.

b) (10%) Find the value of  $\alpha$  (this should be a number).

c) (5%) What is the variance of consumption growth?

Now you are told that  $y_2 = 210, y_1 = 200$  and  $y_0 = 200$ . (This holds for the remaining questions.)

d) (5%) What is  $\Delta C_2$  ?

e) (10%) Assume that the consumer has assets  $A_2 = 0$  at the beginning of period 2. What is  $C_2$ ?

f) (5%) What is the conditional expectation  $E\{C_3|y_2, y_1, y_0\}$ ?

2. (20% of Midterm 2, January 2011) Assume that income follows the time-series process

$$y_t = 3 + 0.5y_{t-1} + e_t ,$$

where  $e_t$  is white noise with variance 4.

a) Assume that the PIH holds. Find the asymptotic value of the coefficient  $\beta$  in the (OLS) regression

$$c_t = \alpha + \beta y_t + u_t ,$$

where  $c_t$  is (PIH-) consumption and  $u_t$  an error term.

b) If  $x_t$  is white noise measurement error with variance 4, with distribution is independent of that of  $y_s$  (for all  $s$ ) and we define  $z_t = x_t + y_t$ . Assume you estimate the OLS regression

$$c_t = \alpha + \gamma z_t + u_t .$$

What is the asymptotic value of  $\gamma$ ?

The following questions are about the CAPM, which we will cover in class Monday).

**3.** (20% of 2009 midterm 2) Consider the CAPM-model.

a) Let  $X$  be an asset whose payout  $PO_X$  is 200 with probability 0.6 and 0 with probability 0.4. Assume that  $PO_X$  has a covariance with the market return (measured in percent) of 10 and that the variance of the market return is 20 (measured in percent). The expected rate of return of the market is 4 percent. Assume that the safe rate of interest is 2 percent.

a) What is the expected return ( $ER_X$ ) to an investment in  $X$ ?

b) What is the price of asset  $X$  in the initial period?

Now assume that  $Y$  is an asset whose payout  $PO_Y$  is 400 with probability 0.6 and 0 with probability 0.4. Assume that  $PO_Y$  has a covariance with the market return (measured in percent) of 20.

c) What is the expected return ( $ER_Y$ ). (Hint: you can provide the answer easily, if you consider the relation between the payout to  $X$  and the payout to  $Y$ .)

**4.** (7% of second core exam 2003.) Asset A and asset B exist for one period and their returns have identical covariances with the market return. The rate return of asset B has a variance that is twice as large as the variance of the rate of return of asset A. Which asset will—if the CAPM holds—have the highest expected rate of return?