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ECONOMICS 7344 – MACROECONOMIC THEORY II, Spring 2011

Homework 4. Friday February 18. Due Wednesday February 23.
(Note: This is due Wednesday since we lost a day. Emiliano has promised to get the answers back to you Friday, so the material can be included in the first midterm.)

1. Define the lag polynomial $a(L) = a_0 + a_1 L$ and $b(L) = b_0 + b_1 L + b_2 L^2$. (Notice: in the notes, and in class, it is often assumed $a_0 = 1$ and $b_0 = 1$. This is just for simplicity and doesn't matter for any results since you can always re-scale the data and the lag-polynomial to to have the first coefficient being unity (write a(L) as $a_0\, a'(L)$ where the lag polynomial $a'(L) = 1 + \frac{a_1}{a_0} L$ and similarly for $b(L)$). The constant a_0 will not affect the properties of the lag-polynomial that we care about. Also notice: The coefficients are real numbers (occasionally complex numbers) and can be negative or positive, it is therefore purely a matter of taste if you write $a(L) = a_0 + a_1 L$ or $a(L) = a_0 - a_1 L$.)

Assume $a_0 = 1$, $a_1 = -3$, $b_0 = 1$, $b_1 = -7$, and $b_2 = 3$.

i) If $x_{t-1} = 2, x_{t-2} = -2, x_{t-3} = -2$, and $x_{t-4} = 9$, what is $a(L)x_t$? and $b(L)x_t$?

ii) What is $a(L)b(L)$? You *have to* do that by finding $a(L)b(L)x_t$ [for general x_t not the specific realizations given] using the definition that $a(L)b(L)x_t = a(L)[b(L)x_t]$ and simplifying).

Define $a(x) = a_0 + a_1 x$ and $b(x) = b_0 + b_1 x + b_2 x^2$.

iii) Find $a(x)b(x)$ and compare the coefficients with $a(L)b(L)$.

2. Define the polynomials $a(x) = 1 + .2\,x$ and $b(x) = 1 - .5x - .5x^2$ and find the roots [meaning the solution(s) to, say, $a(x) = 0$] in each polynomial. What are the roots of the polynomial $c(x) = a(x) * b(x)$?

3. (24% of midterm 1, Spring 2005) Assume that income follows the AR(1) process

$$y_t = 2 + 0.4y_{t-1} + e_t \quad (*)$$

where e_t is white noise with variance 3.
a) Is this time-series process stable?
b) Assume that y_0 is a random variable. For what values of the mean $E(y_0)$ and the variance $\text{var}(y_0)$ will the time series y_t; $t = 0, 1, 2, ...$ be stationary?
c) What is $E_1 y_3$ if $y_1 = 5$ and $y_0 = 2$?
d) Write the infinite Moving Average model that is equivalent to the AR(1) model (*) [assuming that the process now is defined for any integer value of t]. (Half the points are from getting the correct mean term.)

4. (4% Core Spring 2004) Assume that income follows the AR process

$$y_t = 3 + 2.0\, y_{t-1} + e_t$$

where e_t is white noise.
a) Is this time-series process stable?
b) If $y_0 = 2$, what is $E_0 y_1$?

5. Assume that income follows the ARMA process

$$y_t = 3 + 0.3y_{t-1} + e_t$$

where e_t is white noise.
a) Is this time-series process stable?
b) What is $E_{t-2} y_t$ if $y_{t-2} = 5$ and $y_{t-3} = 10$?
c) What is $E_{t-1} y_t$ if $y_{t-2} = 5$ and $y_{t-3} = 10$?

6. Let

$$x_t \ = \alpha_0 \ + \ u_t \ + \ 0.5 * u_{t-1} + u_{t-2} \ ,$$

where u_t is white noise.
Find the auto-covariances for x_t in terms of σ_u^2 (the variance of u_t).