ECONOMICS 7344, Spring 2010
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HOMEWORK 5. Wednesday March 10, due March 22. (This is a long homework, more like two regular homeworks.)

1. A consumer lives for 3 periods and earns 200\$, 100\$, and X\$ in period 1, 2, and 3 respectively. X is a Normally distributed random variable with mean 400 and variance 2. The consumer has a quadratic utility function and is—in period 1—allowed to freely borrow and lend at an interest rate that equals his or her rate of time preference (to be paid back in period 2). The consumer is not allowed to borrow or lend in period 2. Let C_i be the consumption of the representative consumer in period i. Is $C_1 = E(C_2)$ and is $C_2 = E(C_3)$? Find C_1, C_2, and C_3.
[NOTE: This question is modified from one on the 2004 make-up core exam.]

2. Consider a consumer who maximize the *expected* utility of present and future consumption using the criterion function is

$$U(c_t) + \frac{1}{1+\rho} EU(c_{t+1}) + (\frac{1}{1+\rho})^2 EU(c_{t+2}) + ...$$

Assume that the consumer can choose consumption in periods $t, t+1$, and $t+2$ without any constraint. Now assume that the consumer considers an investment in period t that will pay back in period $t+2$ with a (stochastic) interest rate of $r_{t,t+2}$.
i) Find the corresponding Euler equation linking consumption in period t and in period $t+2$. (Assume that you are considering the same asset.)
ii) Show that the Euler equations for period t (relative to period $t+1$) and for period $t+1$ (relative to period $t+2$) are consistent with what you found in part i). (Hint: You need to express the rate of return from period $t+1$ to $t+2$ as a function of the other rates of return.)

3. A consumer lives for 2 periods and earns $Y_1 = 15\$$, in period 1, and in period 2 he or she earns $Y_2 = 10\$$ with probability 1/2 and $Y_2 = 20\$$ with probability 1/2. The consumer maximizes

$$U(C_1) + \beta\, E_1 U(C_2) \ ,$$

where

$$U(C) = 100C - \frac{1}{2}C^2 \ .$$

a. Assume that the rate of interest is 0 and the discount factor is 1 and find C_1 and C_2 (for each of the period 2 outcomes).

b. What is the solution if the interest rate is 10 percent and the discount factor is still 1?

c. What is the solution if the discount rate is 10 percent and the interest rate is 10 percent?

d. And if the discount rate is 10 percent and the interest rate is 0?

e. What is your answer to part a if instead $Y_2 = 2\$$ with probability 0.5 and $Y_2 = 8\$$ with probability 0.5?

4. Repeat question 3 assuming log utility.

5. Romer 7.11. (7.9 in 2nd edition) (This homework builds on a very famous paper by Lucas and it is important that you try to understand the details—in other words, this will be assumed known for the exams. If you get stuck, see me in my office or email me!)

6. (40% of midterm 2, 2008) Assume that y_t follows the stationary AR(1) process

$$y_t = 200 + 0.2y_{t-1} + u_t$$

where u_t is white noise with variance 2.

a) (5%) Find the mean and variance of y_t.

Now assume that the PIH holds such that $\Delta C_t = \alpha u_t$. Assume the rate of interest is 10%.

b) (10%) Find the value of α (this should be a number).

c) (5%) What is the variance of consumption growth?

Now you are told that $y_2 = 210, y_1 = 200$ and $y_0 = 200$. (This holds for the remaining questions.)

d) (5%) What is ΔC_2 ?

e) (10%) Assume that the consumer has assets $A_2 = 0$ at the beginning of period 2. What is C_2?

f) (5%) What is the conditional expectation $E\{C_3 | y_2, y_1, y_0\}$?