

HOMEWORK 8. Due Monday April 26.

1. Assume that 2 agents live for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. Assume that the endowment of the first agent is  $y_0 = 1, y_1 = 4$  and that the endowment of the second agent in period 0 is  $y_0^* = 2$  and in period 1 his or her endowment is  $y_1^* = 6$  in the “good state”  $g$ . In the “bad state”  $b$  the endowment of the second agent is  $y_1^* = 0$ . Assume that the good state happens with probability  $1/2$ .

Assume each agent maximizes a utility function

$$-C_0^{-1} + E_0 - C_1^{-1} .$$

- i) Find the period 0 prices of the Arrow securities that pays out one unit in the good and the bad state, respectively. (I suggest that you follow Obstfeld-Rogoff and parameterize such that the period 0 price of 1 unit delivered in the good state is  $p^g/(1+r)$  and in the bad state it is  $p^b/(1+r)$  which implies that  $p^g + p^b = 1$  when  $r$  is the safe rate of interest. The price of a unit of period 0 consumption is normalized to 1.)
- ii) Find the safe rate of interest  $r$ .
- iii) Find the value (in terms of period 0 output) of the second (“\*”) agent’s output.
- iv) Find the level of consumption of each of the agents in periods 0 and 1 and both states of the world.
- v) Argue, using words, whether the consumption of agent 1 would increase or decrease (compared to the model above) if the utility function were  $-\frac{1}{2}C_0^{-2} - E_0\frac{1}{2}C_1^{-2}$ . (Try to spell out the main economic intuition.)
- vi) Demonstrate what would happen to the interest rate (i.e., would it go up or would it go down) if world output in period 1 were constant rather than a random variable—assume that the mean value of period 1 output is the same. (If you have trouble with this, you will get points if you argue coherently in intuitive terms what would happen.)
- vii) What would happen to the interest rate if world output in period 0 increased (with no change in period 1). (As before, a mathematical demonstration is perfect, but words can get you most of the points if they are precise.)
- viii) What would happen to the interest rate (going up or down) if the agents discounted second period consumption with positive discount rate instead of with the 0 discount rate used so far? (As before, a mathematical demonstration is perfect, but words can get you most of the points if they are precise.)
- ix) Now change the assumptions and assume that no Arrow securities exists, but that the agents in period 0 can trade a safe asset. Find the safe interest rate in this case (define a bond  $B$  which

pays out 1 unit for sure in period 1 and trades at a price  $1/(1+r)$ ). Use the utility function from v).

[Note, the first order conditions will give you some non-linear equations, that aren't easy to solve by hand. You will get full points for stating the restrictions and first order conditions for the problem.]

x) What is the period 1 consumption of agent 2 in this situation (in each state of the world)? [If you don't find B in ix) set use  $B = .1$ ]

2. **38% of Core Exam, Summer 2007** Assume that 3 agents (or countries) live for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. Let superscripts denote the agents and subscripts indicate the time period. Assume that the endowment of the first agent is  $y_0^1 = 3$ ,  $y_1^1 = 5$  in state A and  $y_1^1 = 1$  in state B. The endowment of the second agent in period 0 is  $y_0^2 = 3$  and in period 1 his or her endowment is  $y_1^2 = 4$  in state "A" and  $y_1^2 = 2$  in state "B". The endowment of the third agent in period 0 is  $y_0^3 = 3$  and in period 1 his or her endowment is  $y_1^3 = 2$  in state "A" and  $y_1^3 = 4$  in state "B". Assume that state A happens with probability 1/2. Assume each agent maximizes a utility function

$$\log(C_0) + E_0 \log(C_1) .$$

- a) (18%) Find the consumption of all agents in both periods and in both states of the world.
- b) (10%) Which agent has the highest consumption? Explain the intuition for why. (Note, you only get points for the intuition in this question. If the intuition is correct, you will get full points even if the answer to part a) is wrong.)
- c) (10%) Find the rate of interest. Explain intuitively why it is negative or positive.
- d) (10%) Now assume that there are no Arrow-Debreu assets in the economy, but that the agents can borrow and lend at the equilibrium interest rate (i.e., a "bond economy.") State the first order conditions that determines the lending/borrowing of each of the agents and the equilibrium interest rate. (Do not try to solve the equations.)