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ECONOMICS 7344 – MACROECONOMIC THEORY II, Spring 2009

Homework 4. Wednesday February 18. Due Monday February 23.

1. (24% of midterm 1, Spring 2005) Assume that income follows the AR(1) process

$$y_t = 2 + 0.4y_{t-1} + e_t \quad (*)$$

where e_t is white noise with variance 3.
a) Is this time-series process stable?
b) Assume that y_0 is a random variable. For what values of the mean $E(y_0)$ and the variance var(y_0) will the time series y_t; $t = 0, 1, 2, ...$ be stationary?
c) What is $E_1 y_3$ if $y_1 = 5$ and $y_0 = 2$?
d) Write the infinite Moving Average model that is equivalent to the AR(1) model (*) [assuming that the process now is defined for any integer value of t]. (Half the points are from getting the correct mean term.)

2. (4% Core Spring 2004) Assume that income follows the AR process

$$y_t = 3 + 2.0\, y_{t-1} + e_t$$

where e_t is white noise.
a) Is this time-series process stable?
b) If $y_0 = 2$, what is $E_0 y_1$?

3. (12% Final Exam 2004) Assume that income follows the ARMA process

$$y_t = 3 + 0.3y_{t-1} + e_t$$

where e_t is white noise.
a) Is this time-series process stable?
b) What is $E_{t-2} y_t$ if $y_{t-2} = 5$ and $y_{t-3} = 10$?

4. Let

$$x_t \ = \alpha_0 \ + \ u_t \ + \ 0.5 * u_{t-1} + u_{t-2} \ ,$$

where u_t is white noise.
Find the auto-covariances for x_t in terms of σ_u^2 (the variance of u_t).

5. (10% of final 2005) Assume that income follows the AR(2) process

$$y_t = 3 + 1.5y_{t-1} + 0.5y_{t-2} + e_t \quad (*)$$

where e_t is white noise.

What is $E_1 y_3$ if $y_1 = 4$ and $y_0 = 2$? (Note, the time indices are 0, 1, and 3).

6. Given the AR(2) process

$$x_t \;=\; 3 \;+\; \frac{5}{6} * x_{t-1} \;-\; \frac{1}{6} * x_{t-2} \;+\; u_t$$

where $Eu_t^2 = 2$. Is this process stable?
a) Assuming that the process is stationary, find the variance of x_t and the first-order auto-covariance.

b) Now assume that you know $x_0 = 2$ and $x_1 = 0$. Find the expected value of x_2, x_3, and x_4 conditional on x_0 and x_1. Also find the variance of x_2, x_3, and x_4 conditional on x_0 and x_1.