

HOMEWORK 9. Due Wednesday April 28.

1. Obstfeld-Rogoff 5.3 (parts a and c).

2. Assume that 2 agents live for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. Assume that the endowment of the first agent is $y_0 = 1, y_1 = 3$ and that the endowment of the second agent in period 0 is $y_0^* = 1$ and in period 1 his or her endowment is $y_1^* = 4$ in the “good state” g . In the “bad state” b the endowment of the second agent is $y_1^* = 3$. Assume that the good state happens with probability $3/4$.

Assume each agent maximizes a utility function

$$-\frac{1}{2}C_0^{-2} + E_0 - \frac{1}{2}C_1^{-2} .$$

- i) Find the period 0 prices of the Arrow securities that pays out one unit in the good and the bad state, respectively. (I suggest that you follow Obstfeld-Rogoff and parameterize such that the period 0 price of 1 unit delivered in the good state is $p^g/(1+r)$ and in the bad state it is $p^b/(1+r)$ which implies that $p^g + p^b = 1$ when r is the safe rate of interest. The price of a unit of period 0 consumption is normalized to 1.)
- ii) Find the safe rate of interest r .
- iii) Find the value (in terms of period 0 output) of the second (“*”) agent’s output.
- iv) Find the level of consumption of each of the agents in periods 0 and 1 and both states of the world.
- v) Argue, using words, whether the consumption of agent 1 would increase or decrease (compared to the model above) if the utility function were $-C_0^{-1} - E_0 C_1^{-1}$. (Try to spell out the main economic intuition.)
- vi) Demonstrate what would happen to the interest rate (i.e., would it go up or would it go down) if world output in period 1 were constant rather than a random variable—assume that the mean value of period 1 output is the same. (If you have trouble with this, you will get points if you argue coherently in intuitive terms what would happen.)
- vii) What would happen to the interest rate if world output in period 0 increased (with no change in period 1). (As before, a mathematical demonstration is perfect, but words can get you most of the points if they are precise.)
- viii) What would happen to the interest rate (going up or down) if the agents discounted second period consumption with positive discount rate instead of with the 0 discount rate used so far? (As before, a mathematical demonstration is perfect, but words can get you most of the points if they are precise.)

ix) Now change the assumptions and assume that no Arrow securities exists, but that the agents in period 0 can trade a safe asset. Find the safe interest rate in this case (define a bond B which pays out 1 unit for sure in period 1 and trades at a price $1/(1+r)$). Use the utility function from v).

[Note, the first order conditions will give you some non-linear equations, that aren't easy to solve by hand. You will get full points for stating the restrictions and first order conditions for the problem.]

x) What is the period 1 consumption of agent 2 in this situation (in each state of the world)? [If you don't find B in ix) set use $B = .1$]