

HOMEWORK 10. For class on Wednesday May 3.

1. (40% of last year's final.) Assume that 2 agents (or countries) live for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. Assume that the endowment of the first agent is $y_0 = 3$, $y_1 = 5$ in state A and $y_1 = 1$ in state B. The endowment of the second agent in period 0 is $y_0^* = 3$ and in period 1 his or her endowment is $y_1^* = 4$ in state "A." In the state "B" the endowment of the second agent is $y_1^* = 2$. Assume that state A happens with probability $1/3$. Assume each agent maximizes a utility function

$$\log(C_0) + E_0 \log(C_1) .$$

- a) (20%) Find the consumption of both agents in both periods and in both states of the world.
- b) (10%) Does one agent consume more than the other? Explain the intuition for why.
- c) (10%) Find the rate of interest. Explain intuitively why it is negative or positive.

2. (35% of Final 2004) Consider the case of the 2 agents, 2 periods, 2 states-of-the-world model of Obstfeld-Rogoff Chapter 5.2 (where agents can trade using a full set of Arrow securities). Assume that both agents have quadratic utility functions $U(C_0) + E_0 U(C_1)$, where $U(C_t) = aC_t + bC_t^2$. Assume that $a = 10$ and $b = -0.5$.

Assume that the endowment of the first agent is $y_0 = 2, y_1 = 2$ and that the endowment of the second agent in period 0 is $y_0^* = 2$ and in period 1 his or her endowment is $y_1^* = 4$ in the "good state" g . In the "bad state" b the endowment of the second agent is $y_1^* = 4/3$. Assume that the good state happens with probability 0.25.

- a) Find the price of the Arrow securities for state g and state b .
- b) Explain intuitively why the price of the Arrow security for one of the states may be higher than for the other state. You need to give two reasons and explain the logic.
- c) Find the safe rate of interest.
- d) Assume that the agents now DO NOT have access to Arrow securities but that they have access to a risk-less bond. Continue assuming that the two agents make up the world. Find the amount of the bond purchased or sold in period 1 by each of the agents and the rate of interest. (Ignore non-negativity constraints for consumption if you encounter any.)
- e) Explain the logic of the answer in d) [even if you didn't solve d)]. You need to explain the logic of why the rate of interest will be positive/negative. (Hint: The answer in this case with a quadratic utility function is different from the one you get with a CRRA utility function.)