

HOMEWORK 8. Due by Wednesday April 6.

1. (This is a question from a final exam I gave at Binghamton.) Consider the Consumption-CAPM (C-CAPM). Assume that there are two states of the economy next year, “good” and “bad”, the good state happens with probability 0.5 (and the bad state with probability 0.5). In the good state aggregate consumption grows 4% and in the bad state it grows 0%. Now consider assets D and E. For these we know the pay-outs. For D the payout is 20 in the bad state and 10 in the good state, while for E the payout is 5 in the bad state and 5 in the good state. Use the C-CAPM as it was derived in the handout. The safe rate of return is 1%.

What would be the price of asset D and asset E?

2. An agent has the utility function  $U(x) = -\exp(-x)$  and wants to maximize the expected utility of a stochastic income stream  $R$ , i.e. the agent wants to maximize  $EU(R)$ .

This exponential utility function is commonly used. The purpose of this exercise is to examine what happens to expected utility when you vary the skewness and kurtosis.

a) Consider 2 random variables  $R_1$  and  $R_2$ . The distribution of  $R_1$  is such that  $R_1 = -9$  with probability 0.1 and  $R_1 = 1$  with probability 0.9. The distribution of  $R_2$  is such that  $R_2 = 9$  with probability 0.1 and  $R_2 = -1$  with probability 0.9. Calculate the mean, variance, skewness, and kurtosis for  $R_1$  and  $R_2$ . Then calculate the expected utility of the agent in the case where the agent’s income stream is  $R_1$  and  $R_2$ , respectively. What do you conclude?

b) Now consider to other income streams  $R_3$  and  $R_4$ . The distribution of  $R_3$  is such that  $R_3 = -1$  with probability 0.5 and  $R_3 = 1$  with probability 0.5. The distribution of  $R_4$  is such that  $R_4 = -10$  with probability 0.005,  $R_4 = 0$  with probability .99, and  $R_4 = 10$  with probability 0.005. Calculate the mean, variance, skewness, and kurtosis for  $R_3$  and  $R_4$ . Then calculate the expected utility of the agent in the case where the agent’s income stream is  $R_3$  and  $R_4$ , respectively. What do you conclude?