ECONOMICS 7344, Spring 2005
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Homework 4. Wednesday February 16, do by Monday February 21 (don't turn in).

1. (Final 2004—12%) Assume that income follows the ARMA process

$$y_t = 3 + 0.3y_{t-1} + e_t$$

where e_t is white noise.
a) Is this time-series process stable?
b) What is $E_{t-2}y_t$ if $y_{t-2} = 5$ and $y_{t-3} = 10$?

2. (Make-up final, 2004—12%) Assume that income follows the ARMA process

$$y_t = 3 + 0.3y_{t-1} + e_t + 2e_{t-1}$$

where e_t is white noise.
a) Is this time-series process stable?
b) What is $E_{t-2}y_t$ if $y_{t-2} = 5$ and $y_{t-3} = 10$?

3. (Midterm 2, 2004, 12%) Let

$$x_t \ = 17 \ + \ u_t \ + \ 3 * u_{t-1} \ ,$$

where u_t is white noise.
a) If u_{-1}=10, and $x_0 = 4$, what is
$E_0(x_1)$ and $E_0(x_2)$?

Given the AR(1) process

$$x_t \ = \ 3 \ + \ 0.5 * x_{t-1} \ + \ u_t$$

b) Is this process stable? (Explain your answer.)
c) Assume u_t has variance 1 and X_0 is given as a stochastic variable having mean 6 and variance 4/3.
Is the time-series $X_0, X_1, X_2, ...$ stationary?

4. (Midterm 1, 2004—12%) Let y_t be a stationary time series, and $\gamma(k)$ the k-th order auto-correlation.
a) Prove that $\gamma(k) = \gamma(-k)$.
b) If $a(L)$ is the lag polynomial $1 + .5 * L$, find the inverse of $a(L)$.

c) If $b(L) = 1 + .3 * L - 2 * L^2$ find $a(L) * b(L)$.

5. (Midterm 1, 2003—20%) Consider the AR model

$$y_t \ = \ 37 \ + \ 0.8y_{t-1} \ + \ u_t \ ,$$

where u_t is iid.
a) Is this model stable?

Now assume that you have the model

$$y_t \ = 13 \ + \ u_t \ + \ 0.2u_{t-1} \ + \ .4u_{t-2} \ .$$

b) Calculate the variance of y_t and all auto-covariances.

c) Consider the model in a). If $y_0 = 10$, what is $E_0 y_1$? What is $E_0 y_2$?

d) Consider the model in b). If $u_0 = 10$, $u_{-1} = 0$, and $u_{-2} = 10$, what what is $E_0 y_1$? What is $E_0 y_2$?