

HOMEWORK 10. Due Monday April 26.

Assume that 2 agents live for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. Assume that the endowment of the first agent is  $y_0 = 1, y_1 = 2$  and that the endowment of the second agent in period 0 is  $y_0^* = 1$  and in period 1 his or her endowment is  $y_1^* = 3$  in the “good state”  $g$ . In the “bad state”  $b$  the endowment of the second agent is  $y_1^* = 1$ . Assume that the good state happens with probability  $1/2$ .

Assume each agent maximizes a utility function

$$-\frac{1}{2}C_0^{-2} + E_0 - \frac{1}{2}C_1^{-2} .$$

- i) Find the period 0 prices of the Arrow securities that pays out one unit in the good and the bad state, respectively. (I suggest that you follow Obstfeld-Rogoff and parameterize such that the period 0 price of 1 unit delivered in the good state is  $p^g/(1+r)$  and in the bad state it is  $p^b/(1+r)$  which implies that  $p^g + p^b = 1$  when  $r$  is the safe rate of interest. The price of a unit of period 0 consumption is normalized to 1.)
- ii) Find the safe rate of interest  $r$ .
- iii) Find the value (in terms of period 0 output) of the second (“\*”) agent’s output.
- iv) Find the level of consumption of each of the agents in periods 0 and 1 and both states of the world.
- v) Argue, using words, whether the consumption of agent 1 would increase or decrease (compared to the model above) if the utility function were  $-C_0^{-1} - E_0 C_1^{-1}$ . (Try to spell out the main economic intuition.)
- vi) Demonstrate what would happen to the interest rate (i.e., would it go up or would it go down) if world output in period 1 were constant rather than a random variable—assume that the mean value of period 1 output is the same. (If you have trouble with this, you will get points if you argue coherently in intuitive terms what would happen.)
- vii) What would happen to the interest rate if world output in period 0 increased (with no change in period 1). (As before, a mathematical demonstration is perfect, but words can get you most of the points if they are precise.)
- viii) What would happen to the interest rate (going up or down) if the agents discounted second period consumption with positive discount rate instead of with the 0 discount rate used so far? (As before, a mathematical demonstration is perfect, but words can get you most of the points if they are precise.)
- ix) Now change the assumptions and assume that no Arrow securities exists, but that the agents in period 0 can trade a safe asset. Find the safe interest rate in this case (define a bond  $B$  which

pays out 1 unit for sure in period 1 and trades at a price  $1/(1+r)$ ). Use the utility function from v).

[Note, the first order conditions will give you some non-linear equations, that aren't easy to solve by hand. You will get full points for stating the restrictions and first order conditions for the problem.]

x) What is the period 1 consumption of agent 2 in this situation (in each state of the world)? [If you don't find B in ix) set use  $B = .1$ ]