

**Comprehensive Exam in Macroeconomic Theory–Procedural Instructions**

- (1) Write your answers only on the paper we will provide.
- (2) We will be distributing a numbered sign-in sheet in a moment. The number next to your signature will be your **student number**.
- (3) Every sheet of paper you turn in to us **must** have your **student number** written at the **top-center** of the sheet and **circled**.
- (4) Every sheet of paper you turn in to us **must** have a **page number** written at the **top-right corner** of the sheet.
- (5) When you have finished, or when it is 1:00 pm (whichever comes first), prepare a cover sheet for your exam. This cover sheet should not have a page number, but must have the following things on it:
  - (a) Your **student number** at the top-center, circled.
  - (b) The phrase “Macroeconomics Comprehensive Exam”.
  - (c) The sentence “My last page is page number X”, where X is your total number of pages of answers.

1. (25%) Consider a growth model with the following production function that has constant elasticity of substitution (CES) between capital (K), and labor (L):

$$Y = F(K, L) = A[aK^\rho + (1 - a)L^\rho]^{1/\rho}$$

where  $0 < a < 1$ , and  $\rho < 1$ . Capital accumulates in the usual way (in per capita form):

$$\dot{k} = sf(k) - (n + \delta)k$$

where  $s$ ,  $n$ , and  $\delta$  are the savings rate, population growth rate and the depreciation rate respectively.

a) Analyze the dynamics for capital per worker,  $k$ . Is there a steady state? Under what conditions (parameter values) there is exogenous/ endogenous growth? Consider two cases: Case 1:  $0 < \rho < 1$ , and Case 2:  $\rho < 1$ . What do these cases mean?

b) Consider adding land to the same production function:

$$Y = F(K, L) = A[a(K^\alpha L^{1-\alpha})^\rho + (1 - a)X^\rho]^{1/\rho}$$

where  $X$  denotes the fixed amount of land. Solve part a) again. How does your answer change? Why?

2. (25%) Consider a slight variation of the Lucas model of population where parents get utility from quantity and quality of their children.  $r = \theta = 0$ . Individuals within a generation have identical preferences. Members of generation  $t$  live for 3 periods: in the first period of life,  $(t - 1)$ , individuals consume a fraction of their parent's 1 unit time endowment. In the beginning of the second period of life,  $(t)$ , individuals make a one-time fertility decision,  $n_t$  and an education decision,  $e_t$  for their children together with the saving decision,  $S_t$ . The preferences (assume log utility) of the member of generation  $t$  are defined over second period's consumption,  $C_t$ , third period consumption,  $C_{t+1}$  (savings done in the second period), and the future income of their children,  $n_t w_{t+1} e_t^\beta$ , where  $w_{t+1}$  is the future wage of a child per unit of human capital and  $e_t^\beta$  is the human capital of each child, where  $0 < \beta < 1$ . Parents choose the number of children,  $n_t$ , the amount of education,  $e_t$ , and the savings,  $S_t$  (or  $C_t$  and  $C_{t+1}$ ). These choices are subject to a constraint on the total amount of time, which is 1. Assuming a fixed time cost,  $0 < v < 1$ , for every child, the time left for the household after the fixed cost is incurred is divided between work to earn a wage income,  $w_t$  and educational investment on their children.

a) Set up the maximization problem of parents (assume 1 parent) and solve it for the optimal values  $n_t, e_t, S_t, C_t, C_{t+1}$ . You can normalize wage to 1 for simplicity.

b) What does results imply for consumption smoothing?

c) Is there a trade off between life-cycle savings and investment in children? What does this imply for the growth process?

3. (4%) For Hall's PIH-model explain what is meant by Excess Smoothness of Consumption?
4. (20%) Testing and estimating models of risk sharing.
- a) Explain the test for perfect risk sharing suggested by Barbara Mace.
  - b) Explain how Sala-I-Martin and Sachs examined the income smoothing role of the federal government in the U.S.
  - c) Explain the relation of the paper by Asdrubali, Sørensen, and Yosha to the above two papers.
  - d) Explain how Backus, Kydland and Kehoe used correlations (which ones?) to examine international risk sharing.
5. (12%) Consider the model with imperfect competition in product markets where every agent is similar as described in Romer Chapter 6.
- a) Explain why imperfect competition leads to inefficiently low output. (You need to be precise, but you do not need to solve the model using equations. Whether you use equations or not, you do need to state in words what is going on.)
  - b) If the elasticity of substitution increases will output increase or decrease? Explain why?
6. (7%) A consumer lives for 3 periods and expects to earn 100\$, 200\$, and 300\$ in period 1, 2, and 3 respectively. The consumer has a quadratic utility function and is—in period 1—allowed to freely borrow and lend at an interest rate that equals his or her rate of time preference. The consumer is not allowed to borrow or lend in period 2. Let  $C_i$  be the consumption of the representative consumer in period  $i$ . Is  $C_1 = E(C_2)$  and is  $C_2 = E(C_3)$ ?
7. (7%) Asset A and asset B exist for one period and their returns have identical covariances with the market return. The rate return of asset B has a variance that is twice as large as the variance of the rate of return of asset A. Which asset will—if the CAPM holds—have the highest expected rate of return?