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July 2, 2003

Comprehensive Examination in Macroeconomics

1. (25%) (You have to solve the production and consumption sides separately for this question.)
a) (Production side) Consider the following model with endogenous technology and population. Population growth depends on the difference between current income per capita, y, and the subsistence level of income per capita, which is fixed at $\bar{y}$.

$$n = \beta(y - \bar{y})$$

Output is produced with labor, L and a fixed amount of land, X.

$$Y \;=\; (AX)^{\alpha}\, L^{1-\alpha}$$

Technology growth depends on output per capita.

$$\dot{A}/A \;=\; \gamma\, y,\; 0 < \gamma < \beta$$

Analyze the model's dynamics and steady state(s). Solve for any steady state values of output per capita.
b) (Consumption side) Consider the two period OLG model where people get utility from consumption and having children. In the first period they work and earn a wage income of W_1, consume and have children. In the second period they consume their savings and interest income. There is no discounting. Interest rate is $r > 0$. S_1 denotes first period saving. Children have a fixed cost of v each. The lifetime utility function is,

$$U \;=\; \log(C_1) \;+\; \log(N) \;+\; \log(C_2)$$

where C_1 is consumption in first period and C_2 is consumption in second period, N is the number of children. Write down the budget constraint and the maximization problem and then solve for optimal levels of C_1, C_2, S_1, N. Is there consumption smoothing? Why? Why not?
c.) Discuss how consumption and production sides can be analyzed together? Suggest ways to combine them in an endogenous growth setup. (You don't need to solve anything here, just write in words or in equations.)

2. (25%) Consider the following small open economy OLG model. The world interest rate is $\bar{r}$. There is no depreciation or population growth. A country is perfectly open to the world capital market. The production function in the country is,

$$Y_t \;=\; K_t^{1/2}\,(e_t\, L_t)^{1/2}.$$

where Y_t is the output at time t, K_t is the capital stock, L_t is the labor supply, and e_t is a measure of efficiency units per worker. The growth of e_t is given by,

$$e_{t+1} \;=\; (1+g)\, e_t\,; \quad g > 0$$

Every period a new generation is born. People live two periods. In the first period they work and consume. In the second they do not work and they consume their savings. There are no bequests. The lifetime utility function is,

$$U \;=\; \log(C_1) \;+\; \log(C_2)$$

where C_1 is consumption in the first period and C_2 is consumption in the second period of life. Solve for steady state and derive an expression showing whether in steady state this country will have positive, negative, or zero net foreign assets.

3. (12%) For Hall's PIH-model explain:
a) What is meant by Excess Smoothness of Consumption?
b) What is meant by Excess Sensitivity of Consumption?
c) Suggest a way to test for Excess Sensitivity of Consumption. (You need to be explicit and write down the equation you would estimate and which test you would perform).

4. (14%) Assume that an economy has a finite number of agents that all have Constant Relative Risk Aversion (CRRA) utility functions and that there is one single good in the economy.
a) State a set of conditions under which a representative agent exists. (You should also state explicitly what is meant by "a representative agent.")
b) Demonstrate mathematically that a representative agent exists under these conditions.

5. (24%) Assume that 2 agents live for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. Assume that the endowment of the first agent is $y_0 = 2, y_1 = 2$ and that the endowment of the second agent in period 0 is $y_0^* = 1$ and in period 1 his or her endowment is $y_1^* = 3$ in the "good state" g. In the "bad state" b the endowment of the second agent is $y_1^* = 1$. Assume that the good state happens with probability 1/2.
Assume each agent maximizes a utility function

$$\log(C_0) \;-\; E_0 \log(C_1)\ .$$

a) Find the safe rate of interest r.
b) Find the level of consumption of each of the agents in periods 0 and 1 and both states of the world. Which agent has a higher level of consumption and why?

Now assume that agents do not have access to Arrow-Debreu securities but can trade in a risk-less bond. Further assume that the two agents are part of a larger world economy, and that the net interest rate in that economy is constant and equal to 0.

c) Now find the level of consumption of each of the agents in periods 0 and 1 and both states of the world. (Hint: denote the amount borrowed by an agent in period 0 by B and find B.) If you solve an equation that has several solutions you have to argue which one of the solutions is valid.

d) Based on your answer in c), or based on what your learned about consumer behavior in class, argue using words whether the interest rate would be positive, negative, or zero in the bond economy if the two agents could not borrow from any outside sources (i.e., if the interest rate was determined as the equilibrium rate that made the markets clear, as in question ii) except with bonds being the only asset). You will get points for the validity of your argument (not from just stating the correct sign of the interest rate).