

**Final Exam, November 28th, 2007–5 questions (100 points).**

1. (10%) Let  $X$  be an  $n$ -dimensional normally distributed variable with mean  $\mu$  and variance  $\Sigma$ . Show that  $(X - \mu)' \Sigma^{-1} (X - \mu)$  is  $\chi^2$ -distributed.

2. (25%) Assume that  $Z = (Z_1, Z_2, Z_3, Z_4)'$  is a vector normally distributed random variable with mean  $\mu$  and variance-covariance matrix  $\Sigma$ , where

$$\Sigma = \begin{pmatrix} 4 & 0 & 1 & 0 \\ 0 & 4 & 1 & 1 \\ 1 & 1 & 6 & 2 \\ 0 & 1 & 2 & 4 \end{pmatrix} \quad \text{and} \quad \mu = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

a) What is the conditional mean of  $Z_1$  given  $Z_2$ ?

b) What is the conditional mean of  $Z_1, Z_3$  given  $Z_2$ ?

c) What is the conditional variance of  $(Z_1, Z_2)$  given  $Z_3$ ?

d) What is the conditional mean of  $(Z_1, Z_3)$  given  $(Z_2, Z_4)$ ?

e) What is the distribution of  $Y = 0.25 * [Z_1^2 + (Z_4 - 3)^2]$ ?

3. (15%) a) Prove that  $P(A \cup B) = P(A) + P(B) - P(A \cap C)$ . (You may want to use a Venn diagram.)

b) State and prove Bayes' Law. (You will get 3 points for just stating it.)

4. (20%) a) Derive the Moment Generating Function for the Poisson distribution.

b) Derive the mean for the Poisson distribution. (If you just state the mean you will get 2 points. You may want to use the result from part a but you don't have to.)

5. (30%) Consider a random sample of men and women. 40% of the sample are women. The probability that a man is pro free trade is 30% and the probability a woman is pro free trade is 50%. Some of the the individuals in the sample are economists. (The probability that an economist if pro free trade is, of course, 100%). 10% of the sample are female economists and 10% of the sample are male economists.

1) What is the probability of a randomly selected person being an economist?

2) Assume you select a woman. What is the probability that she is an economist?

3) Again select a woman. You find out that she is pro free trade. What is the probability that she is an economist?

4) Assume you draw (pick) 5 individuals according to these probabilities (each draw has the same probability and the draws are independent). What is the expected number of women?

5) In the situation where you draw 5 individuals as in the previous question, what is the expected number of pro free trade individuals?