ECONOMICS 6331, Fall 2006

Final Exam, December 6th, 2006–6 questions (100 points).

1. (15%) Let a continuous random variable X have density $f(x)$. Let $h(x)$ be a monotone strictly increasing function.
a) What is the density for $Y = h(X)$?
b) Derive the formula you stated in part a).

2. (20%) Assume that X_n is exponentially distributed with density ne^{-nx}.
a) Show that X_n converges in probability to 0.
b) What is the limiting distribution of $\sqrt{n}\,X_n$? (I.e., to what distribution does X_n converge in distribution?)

3. (15%) Assume that X is log-normally distributed and that $\log(X)$ has mean μ and variance σ^2.
a) What is the mean of X.
b) Let $W = X^2$. Find the distribution of W.

4. (24%) Assume that $Z = (Z_1, Z_2, Z_3, Z_4)'$ is a vector normally distributed random variable with mean μ and variance-covariance matrix Σ, where

$$\Sigma = \begin{pmatrix} 4 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 4 & 2 \\ 0 & 1 & 2 & 6 \end{pmatrix} \quad \text{and} \quad \mu = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 6 \end{pmatrix}$$

a) What is the conditional mean of Z_1 given Z_2?
b) What is the conditional mean of Z_1 given (Z_2, Z_4)?
c) What is the conditional variance of (Z_1, Z_2) given Z_3?
d) What is the distribution of $Y = 2\,Z_1 - 2\,Z_2 + Z_3 - Z_4$?

5. (16%) Assume that we have 4 random variables, X_1, X_2, X_3 and X_4 which are independent standard normally distributed variables (with mean 0 and variance 1).
a) What is the probability that exactly 2 of the 4 random variables are positive?
b) What is the probability that $X_1 > X_2$? (Of course, you also need to explain how you get your answer.)

6. (10%) Demonstrate that $P(A \bigcup B \bigcup C) = P(A) + P(B) + P(C) - P(A \bigcap B) - P(A \bigcap C) - P(B \bigcap C) + P(A \bigcap B \bigcap C)$. You may want to use a Venn diagram.