ECONOMICS 6331, Fall 2005

Final Exam, December 2nd, 2005–6 questions (100 points). Sub-questions carry equal weight

1. (30%) Let the random variable X follow a uniform distribution on the interval from 4 to 9.
a) What is the Cumulative Density Function (CDF)?
b) What is the density function (PDF)?
c) Find the CDF for $Y = 2 + 3X$.
d) Find the variance of X and the variance of Y.
e) Let $W = X^2$. Find the CDF for W.
f) Find the PDF for W.

2. (12%) Let X be Binomially distributed with parameters n and p. Find the moment generating function for X.

3. (24%) Assume that Z is a normally distributed random variable with variance 9 and mean 2, and that Z is independent of (X, Y) where (X, Y) is a bivariate normally distributed random variable with mean $\mu' = (0, 0)$ and variance-covariance matrix

$$\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

a) What is the conditional mean of $Y|X$?
b) What is the conditional variance of (X, Z) given Y?
c) What is the conditional mean of X given (Y, Z)?
d) What is the distribution of $2X^2 - 2XY + Y^2$?

4. (12%) Assume you roll two "dice" one blue and one red. To obtain less clutter assume that each "die" can have 1,2,3, or 4 eyes, each outcome occurring with probability 0.25. Let X be the number of eyes on the blue die and Y the number of eyes on the red die.
a) Find the distribution of $Z = X + Y$.
b) Find the joint distribution of Z and X.

5. (10%) Assume that X conditional on a random variable Y is normally distributed with mean $2 + 3Y$ and variance 3. Assume that Y is a stochastic variable who density is $\frac{1}{3} \exp -\frac{y}{3}$ for $y > 0$. What is $E(X)$ and what is $\text{var}(X)$?

PLEASE TURN OVER

6. (12%) Imagine that we select persons associated with either UH or A&M, and imagine that each person can be classified as either liberal or conservative. Use the following (made up) probabilities: The probability that a person selected from UH is a liberal is 80%, and the probability that a person selected from A&M is liberal is 30%.
Now assume you examine 5 people from UH and 4 people from A&M. The attitudes of different persons are independent of each other. Let X be the overall number of liberals.

1) What is the expected number of liberals, $E(X)$?
2) What is the variance, $\text{var}(X)$?