ECONOMICS 6331, Fall 2004

Final Exam, December 1st, 2004—7 questions (101 points). Sub-questions carry equal weight unless otherwise stated.

1. (20%) Consider an exponential distribution with mean θ.
a) What is the Cumulative Density Function (CDF)?
b) What is the density function (PDF)?
c) Find the Moment Generating Function.
d) Find the variance of X. (You need to find it, just stating the variance is not a valid answer).

2. (14%) If X is uniformly distributed on the interval from -10 to 2, and Y is uniformly distributed on the interval from -1 to 1, and X and Y are independent.
a) What is the probability that $max(X, Y)$ (largest value of X and Y) is larger than 0?
b) Write down the joint CDF for X, Y.

3. (12%) Assume $X \sim \chi^2(9)$.
a) What is $E(X)$?
b) Derive the formula for the variance of a $\chi^2(k)$ (chi-square with k degrees of freedom) random variable.

4. (10%) a) State the formula for $P(A \bigcup B)$ in terms of $P(A),\ P(B),\ P(A \bigcap B)$.
b) Prove the formula that you just stated.

5. (15%) Assume that $X_1, X_2, ...$ are independent, identically distributed random variables with mean μ and finite variance σ^2. Let $\overline{X}_n = \frac{1}{N} \Sigma_{i=1}^{N} X_i$.
Prove that $\overline{X}_n$ converges to μ in probability.

6. (15%) Assume $X_1, X_2, ..., X_n$ are independently normally distributed with the mean of $X_i = \mu_i$ and the variance of $X_i = \sigma^2$ for all i.
a) (5%) Write down the formula for the unbiased estimator s^2 of the variance σ^2.
b) (10%) Show that s^2 is a consistent estimator for σ^2.

7. (15%) Let X denote the number of tornadoes observed in Texas during a 12 hour period. Assume that weather conditions are unchanged during that period and that the probability of observing a tornado is constant and independent of how many (if any) were observed previously. If the probability of observing zero tornadoes is p, what is the probability of observing 3 tornadoes?