

Final Exam, December 1st, 2004—7 questions (101 points). Sub-questions carry equal weight unless otherwise stated.

1. (20%) Consider an exponential distribution with mean θ .
 - a) What is the Cumulative Density Function (CDF)?
 - b) What is the density function (PDF)?
 - c) Find the Moment Generating Function.
 - d) Find the variance of X . (You need to find it, just stating the variance is not a valid answer).

2. (14%) If X is uniformly distributed on the interval from -10 to 2 , and Y is uniformly distributed on the interval from -1 to 1 , and X and Y are independent.
 - a) What is the probability that $\max(X, Y)$ (largest value of X and Y) is larger than 0 ?
 - b) Write down the joint CDF for X, Y .

3. (12%) Assume $X \sim \chi^2(9)$.
 - a) What is $E(X)$?
 - b) Derive the formula for the variance of a $\chi^2(k)$ (chi-square with k degrees of freedom) random variable.

4. (10%)
 - a) State the formula for $P(A \cup B)$ in terms of $P(A)$, $P(B)$, $P(A \cap B)$.
 - b) Prove the formula that you just stated.

5. (15%) Assume that X_1, X_2, \dots are independent, identically distributed random variables with mean μ and finite variance σ^2 . Let $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$.
Prove that \bar{X}_n converges to μ in probability.

6. (15%) Assume X_1, X_2, \dots, X_n are independently normally distributed with the mean of $X_i = \mu_i$ and the variance of $X_i = \sigma^2$ for all i .
 - a) (5%) Write down the formula for the unbiased estimator s^2 of the variance σ^2 .
 - b) (10%) Show that s^2 is a consistent estimator for σ^2 .

7. (15%) Let X denote the number of tornadoes observed in Texas during a 12 hour period. Assume that weather conditions are unchanged during that period and that the probability of observing a tornado is constant and independent of how many (if any) were observed previously. If the probability of observing zero tornadoes is p , what is the probability of observing 3 tornadoes?