ECONOMICS 6331, Fall 2003
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Final Exam, December 3rd, 2003—7 questions. All sub-questions carry equal weight.)

1. (18%) Consider two random variables X and Y. Assume they both are discrete and that both X and Y can take the values 1,2, and 3. The probabilities for (X,Y) are shown in the following table:

	X=1	X=2
Y=1	1/12	2/12
Y=2	1/12	2/12
Y=3	2/12	4/12

i) Find the marginal probabilities of X.
ii) Find the mean and the variance of X.
iii) Are the events X = 1 and Y = 1 independent events?
iv) Are the random variables X and Y independent?
v) Find the probability $P(\{X > 1\} \bigcap \{Y \leq 2\})$
vi) Find the conditional distribution of X given $Y = 2$.

2. (12%) Assume X_1, X_2, and X_3 are identically and independently exponentially distributed with mean 1. Let Y be the largest of these 3 random variables ($Y = max\{X_1, X_2, X_3\}$). Derive the density (PDF) for Y.

3. (12%) Assume $X \sim N(0, 9)$, $Y \sim N(2, 9)$, and $Z \sim N(2, 9)$. Further assume that the co-variance between X and Y is 2, while both X and Y are independent of Z.
i) What is $E(X|Y = 2, Z = 3)$? (State the formula you use and then the number.)
ii) What is the conditional variance $Var(X|Z = 3)$?

4. (20%) Assume $X_1, X_2, ..., X_n$ are all iid normally distributed with mean 0 and variance σ^2.
i) State and derive the distribution of the average $\overline{X}$?
ii) State and derive the distribution of s^2.
iii) Normalize $\overline{X}$ with something [you need to state what, I will call it W for now].
such that you get a t-distribution. What are the degrees of freedom?
iv) Demonstrate that $\overline{X}/W$ [where you explained in part iii) what W is] is t-distributed.

5. (12%) Prove the law of iterated expectations (you can do the discrete or the continuous case).

CONTINUED ON REVERSE

6. (16%) In some random experiment, $\hat{\theta}_n$ is a consistent estimator of θ.
i) Is $\log \hat{\theta}_n$ a consistent estimator of $\log \theta$?

Assume X_n is a sequence of random variables which converges in distribution to X.
ii) Is $\theta_n \, X_n$ a consistent estimator of $\theta \, X$ (why or why not)?

7. (10%) Formulate and derive Bayes' Law.