

ECONOMICS 6331, Fall 2006

1. (12% of Final 2003) Assume $X_1, X_2,$ and X_3 are identically and independently exponentially distributed with mean 1. Let Y be the largest of these 3 random variables ($Y = \max\{X_1, X_2, X_3\}$). Derive the density (PDF) for Y .

2. (12% of Final 2003) Assume $X \sim N(0, 9), Y \sim N(2, 9),$ and $Z \sim N(2, 9)$. Further assume that the covariance between X and Y is 2, while both X and Y are independent of Z .
 - i) What is $E(X|Y = 2, Z = 3)$? (State the formula you use and then the number.)
 - ii) What is the conditional variance $Var(X|Z = 3)$?

3. (20% of Final 2003) Assume X_1, X_2, \dots, X_n are all iid normally distributed with mean 0 and variance σ^2 .
 - i) State and derive the distribution of the average \bar{X} ?
 - ii) State and derive the distribution of s^2 (It is a full answer if you state the distribution of some constant times s^2 [of course, you need to specify what the constant is].)
 - iii) Normalize \bar{X} with something [you need to state what, I will call it W for now] such that you get a t-distribution. (What I mean is simply that you write down the expression for the t-statistic—an expression which contains \bar{X} .) What are the degrees of freedom?
 - iv) Demonstrate that \bar{X}/W [where you explained in part iii) what W is] is t-distributed.

4. (16% of Final 2003) In some random experiment, $\hat{\theta}_n$ is a consistent estimator of θ .
 - i) Is $\log \hat{\theta}_n$ a consistent estimator of $\log \theta$?

Assume X_n is a sequence of random variables which converges in distribution to X .

 - ii) Is $\theta_n X_n$ a consistent estimator of θX (why or why not)?

5. (15%) Assume X_1, X_2, \dots, X_n are independently normally distributed with the mean of $X_i = i$ [e.g., the mean of X_2 is 2] and the variance of $X_i = \sigma^2$ for all i . Demonstrate that the estimate s^2 of the variance is a consistent estimator for σ^2 .

6. Ramanathan Exercise 6.2, page 139.

7. Ramanathan Exercise 7.5, page 160.

8. Ramanathan Exercise 7.12, page 161.