

Midterm Exam - October 3, 2016

Each sub-question in the following carries equal weight.

1. (20%) Assume you are estimating a model using Maximum Likelihood with the log-likelihood function written in the common fashion as the sum of the contributions from each observation. Explain how we can estimate the variance of the estimates numerically in two different ways.

2. (20%) Consider the exponential distribution

$$f(x) = \frac{1}{\beta} \exp(-x/\beta) .$$

1) Find the ML estimator.

2) Find the information “matrix” analytically and explain how you can estimate the variance of your estimate using this.

3. (20%) On the next page, I have reproduced a Gauss program that you used for a homework. Write down what should be in the line that I have removed (the contribution to the log-likelihood of the first observation).

4. (20%) 1) Write down the latent-variable model used to derive the Probit model.

2) Write down the log-likelihood function for the Probit model.

5. (20%) 1) (15%) For the standard panel data model, show why the simultaneous inclusion of fixed effects and lagged endogenous variables lead to biased coefficients. (I am not asking you to derive the somewhat complicated formula for the bias, only to show that there will be bias.)

2) (5%) In which cases would we expect this bias to be negligible?

```

new ;
library optmum;

T = 50;
alpha = .5;
sigma = 2;
mu=3 ;
clear lvec ;

/*generate an AR(1) */

u=sigma*rndn(T,1);
x=zeros(T,1) ;

x[1]=mu/(1-alpha)+u[1]/sqrt(1-alpha^2) ;

s=2;
do while s<t+1 ;
    x[s]=mu+alpha*x[s-1]+u[s];
    s=s+1 ;
endo ;

proc logl(b) ;
lvec=loglvec(b) ;
retp(-sumc(lvec)) ;
endp ;

proc loglvec(b);
    local r,L,sigma2,mu,alpha;
mu=b[1] ;
alpha=b[2] ;
sigma2=b[3] ;
L=zeros(t,1);
L[1]=PLEASE WRITE DOWN WHAT SHOULD GO INTO THIS LINE
r=2 ;
do while r<(t+1) ;
    L[r]=-0.5*ln(2*pi) - 0.5*ln(abs(sigma2^2)) - 0.5*(x[r]-mu-alpha*x[r-1])^2/sigma2^2;
    r=r+1;
endo ;
retp(L);

```

```
endp;

/*starting values*/
let sv = 3 .1 2 ;

{b_ml,f,g,retcode} = optmum(&logl,sv);

"estimated parameters:"
"mu-hat:" b_ml[1] ;
"true mu is:" mu;
"alpha-hat:" b_ml[2] ;
"true alpha:" alpha ;
"sigma-hat:" b_ml[3] ;
"true sigma is:" sigma;
```