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Econometrics 1 McElhinney 211 D
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Midterm Exam

Write your answers on **one** side of the blank white paper that I have given you. Do not write your answers on this exam. **You must explain your answers.** If you are confused about a question or you think it is unclear, please ask for clarification before answering**. When testing a hypothesis, make sure to write down the null and alternative, the critical value(s), the test statistic, and your decision (reject or fail to reject).** Use tests with 5% size. Unless otherwise specified, use 2-sided alternative hypotheses.

1. Statistical Properties of the OLS Estimator

Consider the classical linear regression model $y = X\beta + u$, with deterministic regressors and $u \sim (0, \sigma^2 I_n)$.

a. Derive the mean and covariance matrix of $\hat{\beta}_{OLS}$. **(5 points)**

b. Is $\hat{\beta}_{OLS}$ efficient? Be explicit. **(5 points)**

Now assume that the error terms are Gaussian; $u \sim N(0, \sigma^2 I_n)$.

c. Does your answer to part b change? **(2 points)**

d. Consider the statistic, $\frac{\left(R\hat{\beta} - r\right)'\left[R(X'X)^{-1}R'\right]^{-1}\left(R\hat{\beta} - r\right)}{\sigma^2}$, which we know has a χ_j^2 distribution in finite samples, but is not feasible since σ^2 is unknown. Discuss how to deal with this nuisance parameter problem to get the feasible *F*-statistic. **(10 points)**

e. Write down (*i.e.* do not derive) the *F*-statistic in terms of restricted and unrestricted sums of squared residuals and *R*-squares. **(5 points)**

2. Numerical Properties of the OLS Estimator

Consider the OLS decomposition of y into its explained and unexplained components:

$$y = \hat{y} + \hat{u} .$$

a. Write $\hat{y}$ and $\hat{u}$ in terms of projection matrices. **(5 points)**

b. Interpret these projection matrices. **(5 points)**

c. In what sense is the OLS decomposition an orthogonal decomposition? Be specific. **(5 points)**

3. Consider the partitioned regression model:

$$y = X_1\beta_1 + X_2\beta_2 + u .$$

The Frisch-Waugh theorem tell us that we can write the OLS estimator for β_1 as

$$\hat{\beta}_1 = \left(X_1^{*}{}' X_1^{*}\right)^{-1} X_1^{*}{}' y^{*}$$

or

$$\hat{\beta}_1 = \left(X_1' M_2 X_1\right)^{-1} X_1' M_2 y$$

where $M_2 = I - X_2(X_2' X_2)^{-1} X_2'$, $y^{*} \equiv M_2 y$, and $X_1^{*} \equiv M_2 X_1$

a. Interpret M_2, y^{*}, and X_1^{*}. **(15 points)**

b. Interpret $\hat{\rho}$ in the Frisch-Waugh context when the regression model is

$$y_t = a + bt + \rho y_{t-1} + u_t . \quad \textbf{(10 points).}$$

Hint: Think of X_1 and X_2 as

$$X_1 = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{T-1} \end{bmatrix}, \quad X_2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ \vdots & \vdots \\ 1 & T \end{bmatrix}.$$

4. Consider the following two regression models:

$$y = X_1\alpha_1 + M_1 X_2 \beta_2 + u$$

$$y = M_1 X_2 \beta_2 + v$$

Demonstrate that these two regressions must yield the same estimates of β_2. **(10 points)**

(This is problem #6 from ps2)

5. Consider the following regression model with Normal errors:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

where $u_i \sim iidN(0, \sigma^2)$.

a. Test the following hypothesis: **(10 points)**

$$H_0 : \beta_0 = 0$$
$$H_1 : \beta_0 \neq 0$$

b. Test the following hypothesis using a confidence interval: **(10 points)**

$$H_0 : \beta_2 + 2\beta_3 = 8$$
$$H_1 : \beta_2 + 2\beta_3 \neq 8$$

c. Test the following hypotheses: **(10 points)**

$$H_0 : \beta_1 = 0, \beta_2 = 0, \beta_3 = 0$$
$$H_1 : \beta_1 \neq 0, \beta_2 \neq 0, \beta_3 \neq 0$$

Eviews Regression Output for Question 5

Dependent Variable: Y
Method: Least Squares
Date: 03/06/06   Time: 14:27
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.445737			
X1	0.892575	0.171089	5.217019	0.0003
X2	2.361413	0.724042	3.261432	0.0076
X3	3.155499	0.149056	21.16995	0.0000

R-squared	0.995800	Mean dependent var	-6.154827
Adjusted R-squared	0.994654	S.D. dependent var	18.11850
S.E. of regression	1.324742	Akaike info criteri	3.623492
Sum squared resid	19.30437	Schwarz criterion	3.812305
Log likelihood	-23.17619	F-statistic	
Durbin-Watson stat	1.483394	Prob(F-statistic)	

Coefficient Covariance Matrix

	C	X1	X2	X3
C		0.010335	-0.003392	0.002504
X1	0.010335	0.029271	-0.026940	-0.019129
X2	-0.003392	-0.026940	0.524237	-0.023940
X3	0.002504	-0.019129	-0.023940	0.022218

Dependent Variable: Y
Method: Least Squares
Date: 03/06/06   Time: 14:29
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
X1	0.858691	0.171991	4.992650	0.0003
X2	2.372533	0.737768	3.215823	0.0074
X3	3.147291	0.151736	20.74188	0.0000

R-squared	0.995242	Mean dependent var	-6.154827
Adjusted R-squared	0.994449	S.D. dependent var	18.11850
S.E. of regression	1.349967	Akaike info criteri	3.614893
Sum squared resid	21.86891	Schwarz criterion	3.756503
Log likelihood	-24.11170	Durbin-Watson stat	1.274351

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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 03/06/06   Time: 14:29
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.090116   2.278498   0.039550   0.9691
        X1           3.609472   0.699869   5.157357   0.0002
        X2           5.761485   4.367186   1.319267   0.2117
============================================================
R-squared            0.824668    Mean dependent var-6.154827
Adjusted R-squared   0.795446    S.D. dependent var 18.11850
S.E. of regression   8.194568    Akaike info criteri7.221677
Sum squared resid    805.8114    Schwarz criterion  7.363287
Log likelihood      -51.16257    F-statistic        28.22080
Durbin-Watson stat   2.573820    Prob(F-statistic)  0.000029
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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 03/06/06   Time: 14:29
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.461016   0.495082   0.931191   0.3701
        X1           1.013924   0.224238   4.521646   0.0007
        X3           3.263335   0.195164   16.72102   0.0000
============================================================
R-squared            0.991738    Mean dependent var-6.154827
Adjusted R-squared   0.990361    S.D. dependent var 18.11850
S.E. of regression   1.778848    Akaike info criteri4.166665
Sum squared resid    37.97159    Schwarz criterion  4.308275
Log likelihood      -28.24999    F-statistic        720.2144
Durbin-Watson stat   2.269623    Prob(F-statistic)  0.000000
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Dependent Variable: Y
Method: Least Squares
Date: 03/06/06 Time: 14:29
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.130577	0.649137	0.201155	0.8439
X2	3.182884	1.261195	2.523705	0.0267
X3	3.738812	0.175909	21.25429	0.0000

R-squared	0.985407	Mean dependent var	-6.154827
Adjusted R-squared	0.982975	S.D. dependent var	18.11850
S.E. of regression	2.364127	Akaike info criteri	4.735551
Sum squared resid	67.06915	Schwarz criterion	4.877161
Log likelihood	-32.51663	F-statistic	405.1505
Durbin-Watson stat	1.650452	Prob(F-statistic)	0.000000

Dependent Variable: Y
Method: Least Squares
Date: 03/06/06 Time: 14:29
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.098137	2.342481	0.041894	0.9672
X1	4.159123	0.578139	7.193978	0.0000

R-squared	0.799238	Mean dependent var	-6.154827
Adjusted R-squared	0.783795	S.D. dependent var	18.11850
S.E. of regression	8.424714	Akaike info criteri	7.223782
Sum squared resid	922.6854	Schwarz criterion	7.318189
Log likelihood	-52.17836	F-statistic	51.75332
Durbin-Watson stat	2.469233	Prob(F-statistic)	0.000007

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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 03/06/06   Time: 14:29
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C          -3.432049   3.745589  -0.916291   0.3762
        X2           19.16955   6.046461   3.170375   0.0074
============================================================
R-squared            0.436040    Mean dependent var-6.154827
Adjusted R-squared   0.392658    S.D. dependent var 18.11850
S.E. of regression   14.12013    Akaike info criteri8.256646
Sum squared resid    2591.914    Schwarz criterion  8.351052
Log likelihood      -59.92484    F-statistic        10.05128
Durbin-Watson stat   1.938597    Prob(F-statistic)  0.007377
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============================================================
Dependent Variable: Y
Method: Least Squares
Date: 03/06/06   Time: 14:30
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.091573   0.771410   0.118709   0.9073
        X3           4.003574   0.167846   23.85270   0.0000
============================================================
R-squared            0.977661    Mean dependent var-6.154827
Adjusted R-squared   0.975943    S.D. dependent var 18.11850
S.E. of regression   2.810236    Akaike info criteri5.027980
Sum squared resid    102.6666    Schwarz criterion  5.122387
Log likelihood      -35.70985    F-statistic        568.9513
Durbin-Watson stat   2.467139    Prob(F-statistic)  0.000000
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