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Econometrics 1 McElhinney 211 D
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Midterm Exam

Write your answers on **one** side of the blank white paper that I have given you.. Do not write your answers on this exam. **You must explain your answers.** If you are confused about a question or you think it is unclear, please ask for clarification before answering**. When testing a hypothesis, make sure to write down the null and alternative, the critical value(s), the test statistic, and your decision (reject or fail to reject).** Use tests with 5% size. Unless otherwise specified, use 2-sided alternative hypotheses.

1. Statistical Properties of the OLS Estimator

Consider the classical linear regression model $y = X\beta + u$, with deterministic regressors and $u \sim (0, \sigma^2 I_n)$.

a. Derive the mean and covariance matrix of $\hat{\beta}_{OLS}$. **(5 points)**

b. Is $\hat{\beta}_{OLS}$ efficient? Be explicit. **(5 points)**

Now assume that the error terms are Gaussian; $u \sim N(0, \sigma^2 I_n)$.

c. Does your answer to part b change? **(2 points)**

d. Consider the statistic, $\dfrac{(R\hat{\beta} - r)'\left[R(X'X)^{-1}R'\right]^{-1}(R\hat{\beta} - r)}{\sigma^2}$, which we know has a χ^2_j distribution in finite samples, but is not feasible since σ^2 is unknown. Discuss how to deal with this nuisance parameter problem to get the feasible *F*-statistic. **(10 points)**

e. Write down (*i.e.* do not derive) the *F*-statistic in terms of restricted and unrestricted sums of squared residuals and *R*-squares. **(5 points)**

2. Numerical Properties of the OLS Estimator

Consider the OLS decomposition of y into its explained and unexplained components:

$$y = \hat{y} + \hat{u}.$$

a. Write $\hat{y}$ and $\hat{u}$ in terms of projection matrices. **(5 points)**

b. Interpret these projection matrices. **(5 points)**

c. In what sense is the OLS decomposition an orthogonal decomposition? Be specific. **(5 points)**

3. The Frisch-Waugh Theorem

Consider the regression model

$$Y_i = \alpha + \beta X_i + u_i, \tag{1}$$

compared with the model

$$y_i = \beta x_i + u_i, \tag{2}$$

where $y_i \equiv Y_i - \overline{Y}$ and $x_i \equiv X_i - \overline{X}$.

a. Argue that the dependent and independent variables in (2) can be thought of as residuals from (1). What regressions generate these residuals? What is the residual maker which generates these residuals? Interpret this residual maker. **(15 points)**

b. With these in mind, what is the formula for the least squares estimator of the slope coefficient in (1)? **(5 points)**

4. Consider the following regression model with Normal errors:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

where $u_i \sim iidN(0, \sigma^2)$.

a. Construct a 95% confidence interval for β_2. Use this confidence interval to test the following hypothesis: **(5 points)**

$$H_0 : \beta_2 = 4$$
$$H_1 : \beta_2 \neq 4$$

b. Test the following hypothesis using an *F*-statistic: **(5 points)**

$$H_0 : \beta_1 + \beta_3 = 4$$
$$H_1 : \beta_1 + \beta_3 \neq 4$$

c. Test the following hypotheses: **(5 points)**

$$H_0 : \beta_0 = 0, \beta_3 = 0$$
$$H_1 : \beta_0 \neq 0, \beta_3 \neq 0$$

d. Using a p-value, test the null that $\beta_0 = 0$ against the 2-sided alternative. **(5 points)**

Regression Output for Problem 4

```
===============================================================
Dependent Variable: Y
Method: Least Squares
Date: 03/12/08   Time: 15:02
Sample: 1 15
Included observations: 15
===============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
===============================================================
         C           0.445737   0.368727   1.208854   0.2521
        X1           0.892575   0.171089   5.217019   0.0003
        X2           2.361413   0.724042   3.261432   0.0076
        X3           3.155499   0.149056   21.16995   0.0000
===============================================================
R-squared            0.995800    Mean dependent var-6.154827
Adjusted R-squared   0.994654    S.D. dependent var 18.11850
S.E. of regression   1.324742    Akaike info crit   3.623492
Sum squared resid    19.30437    Schwarz criterion  3.812305
Log likelihood      -23.17619    F-statistic        869.2809
Durbin-Watson stat   1.483394    Prob(F-statistic)  0.000000
===============================================================


               Coefficient Covariance Matrix
===============================================================
                  C            X1            X2            X3
===============================================================
     C         0.135960     0.010335    -0.003392     0.002504
     X1        0.010335     0.029271    -0.026940    -0.019129
     X2       -0.003392    -0.026940     0.524237    -0.023940
     X3        0.002504    -0.019129    -0.023940     0.022218
===============================================================
```

```
============================================================
Dependent Variable: Y
Method: Least Squares
Date: 03/12/08   Time: 16:31
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C          -6.154827   4.678176  -1.315647   0.2094
============================================================
R-squared             0.000000    Mean dependent var-6.154827
Adjusted R-squared    0.000000    S.D. dependent var 18.11850
S.E. of regression    18.11850    Akaike info crit   8.696084
Sum squared resid     4595.919    Schwarz criterion  8.743288
Log likelihood       -64.22063    Durbin-Watson stat 2.336955
============================================================
```

```
============================================================
Dependent Variable: Y
Method: Least Squares
Date: 03/12/08   Time: 16:31
Sample: 1 15
Included observations: 15
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.090116   2.278498   0.039550   0.9691
        X1           3.609472   0.699869   5.157357   0.0002
        X2           5.761485   4.367186   1.319267   0.2117
============================================================
R-squared             0.824668    Mean dependent var-6.154827
Adjusted R-squared    0.795446    S.D. dependent var 18.11850
S.E. of regression    8.194568    Akaike info crit   7.221677
Sum squared resid     805.8114    Schwarz criterion  7.363287
Log likelihood       -51.16257    F-statistic        28.22080
Durbin-Watson stat    2.573820    Prob(F-statistic)  0.000029
============================================================
```

Dependent Variable: Y
Method: Least Squares
Date: 03/12/08 Time: 16:31
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
X1	3.601176	0.641539	5.613342	0.0001
X2	5.761946	4.196115	1.373162	0.1929

R-squared	0.824645	Mean dependent var	-6.154827
Adjusted R-squared	0.811156	S.D. dependent var	18.11850
S.E. of regression	7.873600	Akaike info crit	7.088474
Sum squared resid	805.9165	Schwarz criterion	7.182880
Log likelihood	-51.16355	Durbin-Watson stat	2.575678

Dependent Variable: Y
Method: Least Squares
Date: 03/12/08 Time: 16:31
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
X1	0.979464	0.220030	4.451504	0.0007
X3	3.255368	0.193977	16.78224	0.0000

R-squared	0.991141	Mean dependent var	-6.154827
Adjusted R-squared	0.990460	S.D. dependent var	18.11850
S.E. of regression	1.769733	Akaike info crit	4.103100
Sum squared resid	40.71540	Schwarz criterion	4.197507
Log likelihood	-28.77325	Durbin-Watson stat	2.179030

Dependent Variable: Y
Method: Least Squares
Date: 03/12/08 Time: 16:31
Sample: 1 15
Included observations: 15

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.461016	0.495082	0.931191	0.3701
X1	1.013924	0.224238	4.521646	0.0007
X3	3.263335	0.195164	16.72102	0.0000

R-squared	0.991738	Mean dependent var	-6.154827
Adjusted R-squared	0.990361	S.D. dependent var	18.11850
S.E. of regression	1.778848	Akaike info crit	4.166665
Sum squared resid	37.97159	Schwarz criterion	4.308275
Log likelihood	-28.24999	F-statistic	720.2144
Durbin-Watson stat	2.269623	Prob(F-statistic)	0.000000