Credit and Currency

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Preferences and Endowments

- One consumption good which cannot be produced or stored
- The total amount of the good in any given period is N
- There are 2N households divided into two equal types: *odd* and *even*

$$\{y_t{}^o\}_{t=0}^{\infty} = \{1, 0, 1, 0, \ldots\}$$
$$\{y_t{}^e\}_{t=0}^{\infty} = \{0, 1, 0, 1, \ldots\}$$

- Both types maximize

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t^h)$$

- Where $\beta \in (0, 1)$ and $u(\cdot)$ is twice continuously differentiable, increasing, and strictly concave

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A Pareto Optimal Solution

- A social planner has a weighted preference $\theta \in [0, 1]$ for odd agents
- The social planner chooses $\{c_t^o, c_t^e\}_{t=0}^{\infty}$ to maximize:

$$\theta \sum_{t=0}^{\infty} \beta^t u(c_t^o) + (1-\theta) \sum_{t=0}^{\infty} \beta^t u(c_t^e)$$

Subject to:

$$c_t^e + c_t^o = 1, \qquad t \geqslant 0$$

FOC:

$$\theta u'(c_t^o) - (1-\theta) u'(1 - c_t^o) = 0$$

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A Pareto Optimal Solution

- Rearranging:

$$\frac{u'(c_t^o)}{u'(1-c_t^o)} = \frac{1-\theta}{\theta}$$

- Which is time invariant, implying:

Pareto Optimal Solution

$$c_t^o = c^o(\theta)$$
$$c_t^e = 1 - c^o(\theta) = c^e(\theta)$$

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A Competitive Market Solution

- Households take prices $\{q_t^0\}$ as given
- Maximize:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Subject to:

$$\sum_{t=0}^{\infty} q_t^0 c_t \leqslant \sum_{t=0}^{\infty} q_t^0 y_t$$

- Household Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \mu \sum_{t=0}^{\infty} q_t^0 (y_t - c_t)$$

- FOC:

$$\beta^t u'(c_t) = \mu q_t^0, \qquad c_t > 0$$

A Competitive Market Solution

Definition 1

A *competitive equilibrium* is a price sequence $\{q_t^o\}_{t=0}^{\infty}$ and an allocation $\{c_t^o, c_t^e\}_{t=0}^{\infty}$ that have the property that (a) given the price sequence, the allocation solves the optimum problem for households of both types, and (b) $c^e + c^o = 1 \; \forall \; t \geqslant 0$.

- First we need to identify an allocation and price system for which we can verify that the FOC's for both even and odd households are satisfied.
- Start with the Pareto optimal allocation:

$$c_t^o = c^o(\theta)$$
$$c_t^e = 1 - c^o(\theta) = c^e(\theta)$$

A Competitive Market Solution

- Plugging the Pareto allocation into the FOC for odd households yields:

$$q_t^0 = \frac{\beta^t u'(c^o)}{\mu^o}$$

Or,

$$q_t^0 = q_0^0 \beta^t$$

- Normalizing $q_0^0 = 1$ and plugging into budget constraint:

$$\text{Odd}: \quad \sum_{t=0}^{\infty} \beta^t c^o = \sum_{t=0}^{\infty} \beta^t y_t^o$$

$$\text{Even}: \quad \sum_{t=0}^{\infty} \beta^t c^e = \sum_{t=0}^{\infty} \beta^t y_t^e$$

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A Competitive Market Solution

$$\text{Odd}: \quad \frac{c^o}{1-\beta} = \frac{1}{1-\beta^2}$$

$$\text{Even}: \quad \frac{c^e}{1-\beta} = \frac{\beta}{1-\beta^2}$$

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A Competitive Market Solution

Competitive Market Solution

$$c^o = \frac{1}{1+\beta}$$

$$c^e = \frac{\beta}{1+\beta}$$

$$q_t^0 = \beta^t$$

- The competitive market solution is Pareto Optimal

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Ricardian Proposition

- Assume a government which levies taxes τ_t^i
- The government uses the tax revenues to purchase some constant $G \in (0,1)$
- The household's budget constraint then becomes:

$$\sum_{t=0}^{\infty} q_t^0 c_t^i \leqslant \sum_{t=0}^{\infty} q_t^0 (y_t^i - \tau_t^i)$$

- The government's budget constraint is:

$$\sum_{t=0}^{\infty} q_t^0 G = \sum_{i=o,e} \sum_{t=0}^{\infty} q_t^0 \tau_t^i$$

Ricardian Proposition

Definition 2

A *competitive equilibrium* is a price sequence $\{q_t^o\}_{t=0}^{\infty}$, a tax system $\{\tau_t^o, \tau_t^e\}_{t=0}^{\infty}$ and an allocation $\{c_t^o, c_t^e, G_t\}_{t=0}^{\infty}$ such that given the price system and the tax system the following conditions hold: (a) the allocation solves each consumer's optimum problem, and (b) the government budget constraint is satisfied for all $t \geqslant 0$, and (c) $N(c_t^o + c_t^e) + G = N \; \forall \; t \geqslant 0$.

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Ricardian Proposition

- Let $\tau^i \equiv \sum_{t=0}^{\infty} q_t^0 \tau_t^i$
- Then it follows that:

$$c^o = \frac{1}{1+\beta} - \tau^o(1-\beta)$$
$$c^e = \frac{\beta}{1+\beta} - \tau^e(1-\beta)$$

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Ricardian Proposition

Ricardian Proposition

The equilibrium is invariant to changes in the *timing* of tax collections that leave unaltered the present value of lump-sum taxes assigned to each agent.

Loan Market Interpretation

- Define total time t tax collections as $\tau_t = \sum_{i=o,e} \tau_t^i$
- Then the government's budget constraint becomes:

$$(G_0 - \tau_0) = \sum_{t=1}^{\infty} \frac{q_t^0}{q_0^0} (\tau_t - G_t) \equiv B_1$$

Or:

$$\frac{q_0^0}{q_1^0} (G_0 - \tau_0) + (G_1 - \tau_1) = \sum_{t=2}^{\infty} \frac{q_t^0}{q_1^0} (\tau_t - G_t) \equiv B_2$$

Loan Market Interpretation

- Using different notation:

$$R_1 B_1 + (G_1 - \tau_1) = B_2$$

- In general:

$$R_t B_t + (G_t - \tau_t) = B_{t+1}, \qquad t \geqslant 0$$

A Monetary Economy

- Preferences and endowments are the same as above
- Shut down *all* loan markets and rule out intertemporal trades
- Replace complete markets with fiat currency
- At time 0 the government endows each *even* agent with $\frac{M}{N}$ units of unbacked, inconvertible currency
- *Odd* agents are given nothing in time 0
- Let p_t be the price level in time t
- Contemporaneous exchanges of currency for goods are the only transactions allowed

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A Monetary Economy

- Given the price sequence $\{p_t\}_{t=0}^{\infty}$ the household's problem is to choose $\{c_t, m_t\}_{t=0}^{\infty}$ to maximize:

$$\sum_{t=0}^{\infty} = \beta^t u(c_t)$$

Subject to

$$m_t + p_t c_t \leqslant p_t y_t + m_{t-1}, \qquad t \geqslant 0$$

The household Lagrangian is then:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{u(c_t) + \lambda_t (p_t y_t + m_{t-1} - m_t - p_t c_t)\}$$

A Monetary Economy

- The FOC's with respect to c_t and m_t are:

$$\begin{aligned} u'(c_t) &= \lambda_t p_t, \qquad c_t > 0 \\ -\lambda_t + \beta\lambda_{t+1} &= 0, \qquad m_t > 0 \end{aligned}$$

- Substituting,

$$\frac{\beta u'(c_{t+1})}{p_{t+1}} = \frac{u'(c_t)}{p_t}, \qquad m_t > 0$$

A Monetary Economy

Definition 3

A *competitive equilibrium* is an allocation $\{c_t^o, c_t^e\}_{t=0}^{\infty}$, nonnegative money holdings $\{m_t^o, m_t^e\}_{t=-1}^{\infty}$, and a nonnegative price level sequence $\{p_t\}_{t=0}^{\infty}$ such that (a) given the price level sequence and (m_{-1}^o, m_{-1}^e), the allocation solves the optimum problems of both types of households, and (b) $c_t^o + c_t^e = 1,\ m_{t-1}^o + m_{t-1}^e = M/N\ \forall\ t \geqslant 0$.

A Monetary Economy

- Assume the Pareto Optimal solution of constant consumption through time,

$$\{c_t{}^o\}_{t=0}^{\infty} = \{c_0, 1-c_0, c_0, 1-c_0, \ldots\}$$
$$\{c_t{}^e\}_{t=0}^{\infty} = \{1-c_0, c_0, 1-c_0, c_0, \ldots\}$$

- Let $p_t = p$. Then for the odd consumer:

$$\frac{\beta u'(1-c_0)}{p} = \frac{u'(c_0)}{p}$$

Rearranging,

$$\beta = \frac{u'(c_0)}{u'(1-c_0)}$$

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A Monetary Economy

- Because $\beta < 1$ it follows that $c_0 \in (\frac{1}{2}, 1)$
- Notice that c_0 is not constant, rather, it fluctuates through time. This solution is not Pareto Optimal.
- To pin-down the price level, consider the odd agents period 0 budget constraint:

$$pc_0 + M/N = p \cdot 1$$

Or,

$$p = \frac{M}{N(1 - c_0)}$$

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A Monetary Economy

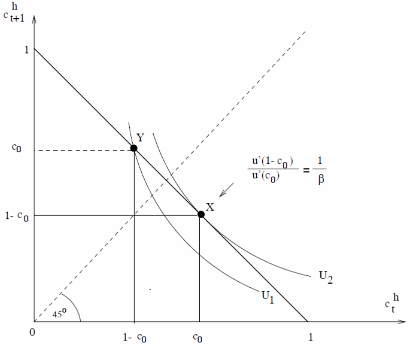

Figure 24.4.1: The tradeoff between time-t and time–$(t+1)$ consumption faced by agent $o(e)$ in equilibrium for t even (odd). For t even, $c^o_t = c_0$, $c^o_{t+1} = 1 - c_0$, $m^o_t = p(1-c_0)$, and $m^o_{t+1} = 0$. The slope of the indifference curve at X is $-u'(c^h_t)/\beta u'(c^h_{t+1}) = -u'(c_0)/\beta u'(1-c_0) = -1$, and the slope of the indifference curve at Y is $-u'(1-c_0)/\beta u'(c_0) = -1/\beta^2$.

Questions?

- Questions?

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