1) What is a **convex** set $C \subseteq \mathbb{R}^n$? What is an **epigraph** of a function $f : C \rightarrow \mathbb{R}$. What is a **convex** function $f : C \rightarrow \mathbb{R}$. What is a **quasi-convex** function $f : C \rightarrow \mathbb{R}$.

2) Let $C$ be a non-empty, open convex set in $\mathbb{R}^n$ and $f : C \rightarrow \mathbb{R}$ be a differentiable function. Prove that $f$ is convex on $C$ if and only if

$$f(y) \geq f(x) + \nabla f(x)^T (y - x), \quad \forall x, y \in C.$$ 

3) Let $f$ be a twice continuously differentiable function in $\mathbb{R}^n$. What are the first-order necessary conditions of optimality? What are the second-order necessary conditions? What are the sufficient conditions?

4) Let $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ be continuously differentiable in the open convex set $D \subset \mathbb{R}^n$, $x \in D$, and let $F'$ be Lipschitz continuous at $x$ in the neighborhood $D$. (We will use a vector norm, the induced matrix norm and the Lipschitz constant $\gamma$.) Then, for any $x + p \in D$, prove that

$$\|F(x + p) - F(x) - F'(x)p\| \leq \frac{\gamma}{2} \|p\|^2.$$ 

5) Write an algorithm for solving the unconstrained optimization problem:

$$\min_{x \in \mathbb{R}^n} f(x)$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is twice continuously differentiable. Use the BFGS method and an inexact line search method.

6) What is a **convex program**? Given a convex program, define the **dual program**.

7) Consider the nonlinear programming problem:

$$(NLP) \min_{x \in \mathbb{R}^n} f(x)$$

subject to $h_i(x) = 0, i = 0, \ldots, p,$

$$g_i(x) \geq 0, i = 0, \ldots, m,$$

where $f, g_i, h_i$ are all twice continuously differentiable from $\mathbb{R}^n$ into $\mathbb{R}$. If $x^*$ satisfies the strong constraint qualification for problem $(NLP)$ state the first order $(KKT)$ and second order necessary conditions for problem $(NLP)$. What are the sufficient conditions?
8) Consider the standard form of the linear program:

\[
(LP) \quad \min_{x \in \mathbb{R}^n} c^T x \\
\text{subject to } Ax = b, \quad x \geq 0,
\]

where $A$ is $m \times n$. State the dual problem. Prove the weak duality theorem. State the strong duality theorem.

9) Consider the problem:

\[
(NEP) \quad \min f(x) \\
\text{subject to } h_i(x) = 0, \ i = 0, \ldots, p,
\]

where $f, h_i$ are all twice continuously differentiable from $\mathbb{R}^n$ into $\mathbb{R}$. Write an algorithm for solving $(NEP)$ using an Augmented Lagrangian method. Write an algorithm for solving $(NEP)$ using sequential quadratic programming.

10) Write a conjugate gradient algorithm for minimizing the quadratic $q(x) = \frac{1}{2}x^TAx - b^Tx$, where $A \in \mathbb{R}^{n \times n}$ is symmetric positive definite and $x, b \in \mathbb{R}^n$. 