Applicable Analysis
Preliminary Examination Syllabus.

Students taking the Applied Analysis preliminary examination are expected to have a thorough knowledge of finite dimensional differential calculus and of metric space topology as in the senior level real analysis sequence. The specific topics that may be examined include the following.

## Contraction mapping theorem and its applications

1. Complete metric spaces, Lipschitz continuous mappings, fixed point iteration and the contraction mapping theorem.
2. The finite dimensional inverse and implicit function theorems
3. Existence-uniqueness theorems for initial value problems for systems of ordinary differential equations.
4. Solvability of Volterra integral equations, and the represent of solutions.
5. Perturbation theory and the dependence of solution of equations on parameters.

## Hilbert spaces and solvability of linear equations

1. Inner products, orthogonality, definitions and examples of real Hilbert spaces.
2. Best approximation theorem, projection theorem and Bessel's inequality.
3. Orthonormal bases and Parseval's equality.
4. Continuous linear functionals, representation theorem and dual spaces.
5. Continuous linear operators, adjoints and continuous bilinear forms.
6. Fredholm splitting theorem and the solvability of linear operator equations, the Fredholm alternative.
7. The Lax-Milgram theorem.

Note that in the above sections the emphasis will be on the applications of the theory to the analysis of equations that typically arise in applications.

## Finite-dimensional Optimization Theory

1. Existence results for minimizers and local minimizers,
2. Analysis of finite-dimensional convex sets and functions,
3. Extremality conditions and necessary, respctively sufficient, conditions for local minimizers.
4. Convex constrained optimization, Lagrange and KKT multipliers.
5. Lagrangians for constrained optimization problems.
6. Applications to the proofs of inequalities and the eigenvalues of real symmetric matrices.

## References.

The following texts treat various topics included in this syllabus.


1. L.D. Berkovitz, *Convexity and Optimization in $R^n$*, Wiley 2002.
2. D.H. Griffel, *Applied Functional Analysis* Dover 2002.
3. C.W. Groetsch, *Elements of Applicable Functional Analysis* M. Dekker, 1980.
4. A.W. Naylor and G.R. Sell, *Linear Operator Theory in Engineering and Science* Springer Verlag 1982.
5. A.L. Peressini, F.E. Sullivan & J.J. Uhl, Jr, *The Mathematics of Nonlinear Programming*, Springer 1988.