Real Analysis Preliminary Examination Syllabus
Department of Mathematics, University of Houston

August, 2011

**TOPOLOGY:** Open and compact sets, Hausdorff spaces, Urysohn’s lemma, Tietze extension theorem.

**METRIC SPACES:** Metrics, metric topology, continuity, the Cauchy condition and completeness.

**MEASURES** Sigma-algebras, Borel sets, outer measures, monotone class theorem, Riesz representation theorem, Borel measures, regularity properties, Lebesgue measure on $\mathbb{R}^n$, signed and complex measures, Jordan decomposition, total variation of a measure, absolute continuity, product measures.

**INTEGRATION:** Measurable functions, simple functions, approximation properties, integration of non-negative functions, integration of complex and real-valued functions, convergence theorems for integrals (e.g., Fatou, monotone, dominated convergence), almost everywhere convergence, Lusin’s theorem, Egoroff’s theorem, the Fubini-Tonelli theorem.

**DIFFERENTIATION:** Bounded variation, absolute continuity, Lebesgue and Radon-Nikodym theorem, differentiation, fundamental theorem of calculus.

**BASICS of FUNCTIONAL ANALYSIS and $L^p$ SPACES:** Normed and Banach spaces, Hahn-Banach theorem, Hilbert spaces, Baire Category theorem, Open Mapping Theorem, Closed Graph Theorem, Principle of Uniform Boundedness, Applications to Fourier series, $L^p$ spaces and their duals, completeness, convergence, density, $C(X)$ spaces and their duals.

**FOURIER TRANSFORM:** Convolutions, Inversion, Plancherel’s identity.

**References:**