Application of Non-Uniform Fast Fourier Transform Methods for More Efficient Acoustic Solvers

Thomas Markovich[1] and Donald J. Kouri [2]
[1] Department of Physics, University of Houston, Houston, TX 77204-5005
[2] Department of Physics, University of Houston, Houston, TX 77204-5005

ABSTRACT

We present a new method to solve the Acoustic Wave Equation with nonuniform gridding by modifying the standard spectral algorithm. Our algorithm, in particular, uses the fast fourier transform at non-equispaced nodes for efficient Acoustic Wave Equation solvers using Finite Differencing propagators in time. We first examined the comparison of standard spectral methods to their NFFT counterparts and found significant advantages for non-uniform grids as well as the applications of the DAF spectrally using nonuniform gridding. Finally, we present propagators in both 1 and 2 dimensions, which showed 7-9% efficiency gains over the traditional approaches.

Motivation

In recent years, researchers have devoted significant amounts of work and attention to time domain solutions for acoustic wave propagation with significant discontinuities in the conductive velocity field. Various methods for the propagator have been developed including spectral, pseudo-spectral polynomial, and finite difference based propagators. However, each method depends on spatial differentiation and as it stands, for discontinuous media and non-uniform gridding, finite differencing is the practical choice because of the ability to apply this stencil quite easily to non-uniform grids. The issue with methods requiring a spatially applied stencil is that their computation time increases with O(n^2) complexity where as spectrally we only increase O(n logn). Because of this Spectral Methods are particularly attractive. The issue comes in that Spectral differentiation require substantial extreme oversampling to achieve comparable accuracy to Finite Difference methods that their efficacy in these problems has been questioned. Recent developments however, have showed that using trigonometric interpolation with proper windowing functions allow for accurate Fast Fourier Transforms (FFTs) on non-uniform grids.

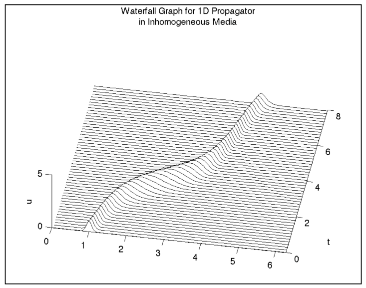
Outline of Method

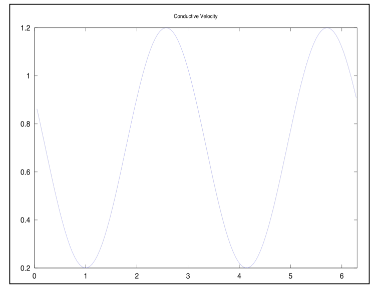
For simplicity, we limit ourselves to the 1-Dimensional case but it should be noted that extension to higher dimensions is relatively simple. In general, we try to use standard FFT libraries (we use FFTW) and apply a windowing function that is an eigenfunction of the Fourier Transform. To approximate the standard trigonometric polynomial, we want to sum a linear combination of shifted 1-period window functions of the form

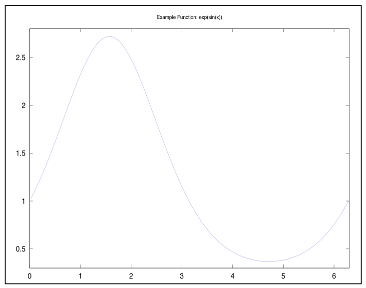
$$S_1(x) := \sum_{l \in I_n} g_l \phi\left(x - \frac{l}{n}\right).$$

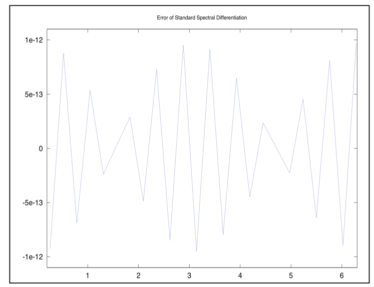
We make the function being approximated by the windowing function of choice to be well behaved and that the Fourier series will be well localized in both space and frequency. The prefered windowing function was the Kaiser-Bessel function because it kept the truncation error small while being well localized in both dimensions.

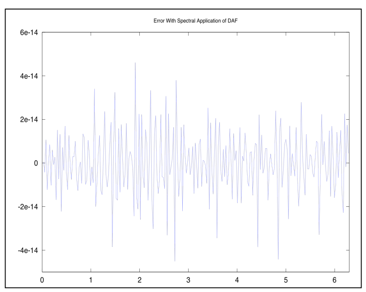
$$\phi(x) = \begin{cases} \frac{\sinh(b\sqrt{m^2-n^2x^2})}{\sqrt{m^2-n^2x^2}} & for\ |x| \le \frac{m}{n} \\ \frac{\sinh(b\sqrt{n^2x^2-m^2})}{\sqrt{n^2x^2-m^2}} & otherwise \end{cases}$$

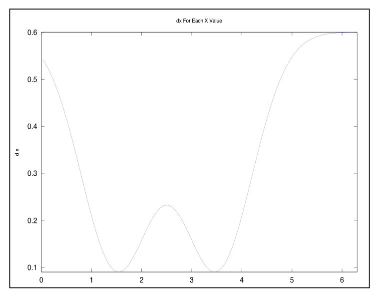


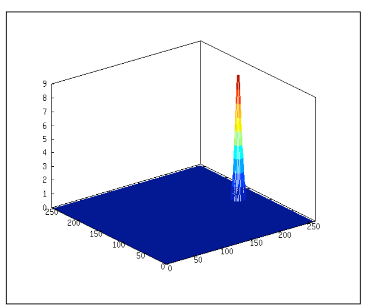


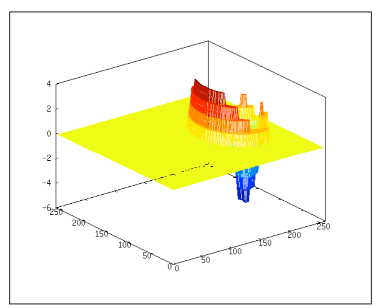












Outline of Method

The fastest way to do this evaluation is through precomputation of the anstantz. Thus, one only has to store the large amount of (2m+1)M real numbers but use no extra flops to during each NFFT.

To transform back from the fourier domain into the spatial domain which is overdetermined, we simply solve the following matrix equation:

$$||y - A\hat{f}||w = \left(\sum_{j=0}^{M-1} w_j |y_j - f(x_j)|^2\right)^{1/2}$$

Additionally; we explored the application of the DAF using FFTs. Because the DAF is a stencil in space and applied through convolution; in the fourier domain, we need only to pointwise multiply the DAF by the FFT of the function.

Discussion and Results

We found a number of useful features:

1)The NFFT algorithm, under the appropriate oversampling conditions proved to be as accurate as the standard spectral FFT methods.

2)Comparisons of the NFFT spatial differentiation involving convolutions showed that the NFFT was at least 7% more efficient and had a maximum of 9% more efficient for similar accuracies.

3)The application of the DAF using spectral methods is preferred. What's more, it appears that the DAF filters some errors in the higher frequencies due to the windowing properties of the DAF.

Further Research

We are now working on extending the NFFT methods to 3D. Additionally, the current propagator employs a standard finite differencing in time approach where the errors propagate through the grid linearly. In the future, we would like to work on an approach using a Pseudo-Spectral approach where errors propagate much more slowly. Additionally, we would like to pursue this NFFT spatial differentiation scheme in other applications such as Molecular Dynamics and other PDE solvers.

References

Keiner, Jens; Kunis, Stevan; Potts, Daniel, NFFT 3.0 – Tutorial
D.K. Hoffman and D. J. Kouri, *Hierarchy of local minimum solutions of Heisenberg's uncertainty principle, (Phys. Rev. A. 85,5263)*