A World of Possibilities

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INTRODUCTION

The title, A *World of Possibilities,* makes reference to both probability and statistics and the fact that they are present in our daily life, in so many ways that often we are not even aware of it. Indeed, the word possibility used in common language, gives us the idea that an event, not being a fact yet, can happen–even though it is not granted that it will. This ambiguity inherent to the word possibility can be clarified if we use another term, probability, which is the formal word used in statistics. When I asked my students to tell me in their own words what probability is, someone said something quite close to the formal definition (indeed, it could well be an alternate definition), saying that "probability is the chance of something to come out." In formal words, probability is "the likelihood that an event will occur, as measured by the relative frequency of the occurrence of events of the same kind" (Barnhart 1988). In this definition the key word is *measured*, because it entails a rationalization of something that can or cannot happen, giving numeric values to the likelihood—or unlikelihood—that an event occurs.

This unit is intended for elementary fourth and fifth graders in their science or mathematics courses. At such grade levels and age, the students are still imbedded in a world of imagination and "the unexplained," and it is—from a teacher's perspective—interesting, and even amusing, to observe the students reaction when you can predict certain outcomes for simple experiments or "tricks" with poker cards. Students are puzzled by the fact that you can "guess" what is going on, and they even can imagine that you possess certain special abilities or even certain "powers." For quite a few elementary students, this will be a satisfactory explanation on how you know what you know! The next step beyond "showing a trick" is to explain to them the logic behind it: when you explain a "trick" you start unveiling the logic behind the phenomenon, and how the outcomes… come out!

UNIT RATIONALE

At this age the students already have the first skills in logical thinking and, by taking their first formal science courses, realize that science, experiments, and discoveries are related to numbers and calculations, which eventually yield answers. They even have already the notion (many times thanks to their fictional heroes) that given enough information (data, some events are predictable (at least to some extent and with a certain degree of accuracy); this is, they have a first notion of probability. What they don't have are the tools.

In Texas, the Houston Independent School District (HISD) has prepared a science curriculum for every grade level known as Project CLEAR (Clarifying Learning to Enhance Achievement Results), which is in alignment with the state curriculum requirements and objectives. Project CLEAR, is currently in use for science in fourth and fifth grades, and is composed of nine units of study at each grade level. Some of the lessons included offer a possibility to discuss statistics and probability; nevertheless, such lessons keep a merely descriptive approach, not discussing statistics any further.

In order to better prepare our pupils, it is desirable to extend or include some lessons in which the students learn specifically about probability and statistics and their connections with other fields in science, acquiring the basic principles and tools necessary for further studying these disciplines in more depth.

UNIT OVERVIEW

This unit aims to illustrate some basic principles of probability and statistics which apply to everyday life situations. The examples of real situations are diverse in nature, and, therefore, this unit will deal with different subjects in probability and statistics prepared on the basis of one topic per lesson.

Through this unit the students will have a first formal approach to probability and statistics which are somehow implicit in certain topics in the elementary curriculum, but which do not appear integrated in one lesson for that particular purpose. The students can acquire and develop the basic logic skills necessary for "thinking probabilistically" as well as the basic mathematical tools to perform basic statistical operations.

The lessons proposed here focus on some statistical and probabilistic aspects of science through a number of activities and demonstrations which constitute the core of the corresponding lessons; such activities will give coherence to the concepts and definitions taught along with the unit. These activities could be used for explaining a number of topics from the first day, but for the sake of clarity, and in order to go from simple to complex, only a few concepts will be dealt with at each session. Most of the hands-on activities can be performed by the students alone, but explanations about the purpose of such activities and how to perform them fairly are needed for the games to be of instructional value and not only chance games.

The first "encounter" with probability will be the subject of the first lesson, which is tossing the coin and determining the number (and eventually the percentage) of heads and tails on a series of throws. Therefore, the probability of one or the other outcome must be inferred from this experience. A further step to understanding probabilities will be tossing a die, calculating the probability of a given number to come out, depending on the number of faces of the die. Nowadays, there are dice with ten, twelve, or more faces, a fact which opens the door to extending the activity. Another activity with dice can be to try to understand and calculate the probabilities of obtaining a given combination of numbers with two dice. Finally, the probability of drawing a ball (or marble) of a certain color from a bag containing several balls will be calculated and tried.

At the end of these lesson(s), the students will understand the concepts of probability, trial, independent events, combined events, conditional events, and mutually exclusive events. They must be able to calculate probabilities of simple games as those described above.

Another exercise will deal with the probability of two persons being born in a certain week, lets say, the first week of the month (regardless of which month), among a group of 2, 4 or 7 people, or a variant of this, calculating the probability of two persons born in the same month.

As mentioned before, some lessons in the curriculum offer the possibility to discuss statistics and probability applied to real situations. In the fourth grade curriculum, unit nine has two lessons that deal in particular with probability and/or statistics. These are lesson two, dealing with inherited trait, and lesson five, dealing with heredity. Such lessons are basic, and there we have an opportunity to extend them or add supplemental lessons. In lesson two, we can introduce the concepts of frequency of occurrence of a given human trait and its likelihood of appearance in a population sample. After investigating the probability of finding a given trait in a population, and repeating the survey for a second trait, we will be able to calculate the combined probability of finding individuals bearing both traits.

This unit has been conceived as the first foot on the floor of probability and statistics for elementary students. My main interest is to explain and teach them, in a simple way, how some probabilities come to light. As a child I was puzzled by the question as to *how people know that this is more likely to happen than this other thing.* And even worse, I simply could not understand how in the world it was possible to adjudicate a number for a probability. *Where does that number come from? What does that mean? Why*? This unit will guide the elementary students to their first understanding and calculation of simple probabilities; it will help them to understand where those numbers come from, what they mean and—in some cases—what to do with them.

BACKGROUND

Tossing the Coin: Lesson One Teacher's Background

In the first lesson the following concepts and definitions are useful:

> Probability is "the likelihood that an event will occur, as measured by the relative frequency of the occurrence of events of the same kind" (Barnhart 1988).
>
> Independent events are those that are not influenced by the previous outcome, nor influence the next outcome, like coin tossing.

Coin tossing is the classical starter, because its simplicity and clarity. As a coin has two sides, if we toss it, it can only fall either head or tail (if it does not happen to stand on its side!), so we divide the result by two, the two sides, or 1/2, which gives each side a 50 % probability of being the actual outcome.

The proof or demonstration of this is seen when we toss a coin a sufficient number of times to realize that about half of the times, the outcome will be heads and the rest will be tails … that is our 50 % possibility of being one or the other! Of course, probabilities are not exact, and it is expected that the percentage of heads will be close to 50% but not exactly. The tails will constitute the complement needed for a 100 % of outcomes.

Throwing the Dice: Lesson Two Teacher's Background

In the second lesson, throwing one and two dice, some useful information is:

Probability space or sample space is the set of all possible results of an experiment (Downing and Clark 1997). An event is a subset of the probability space (Downing and Clark 1997), or in other words, a group of some of the possible outcomes of the probability space. Combination: the order of two or more different elements in an arrangement is not taken into account, AB = BA. Permutation: the order of two or more different elements in an arrangement is taken into account, AB different from BA.

The probability of a union is the added probability that two or more events will occur.

This lesson is just one step further to the coin tossing. A die has six sides; each side has the same probability of ending on top after tossing the die; therefore, the probability of any number to come out is 1/6. When working with only one die, we can understand how the probabilities add up when we consider more than one outcome as a possible result. Let's say that we accept as our result any odd number: the probability of such result is 1/2 because the probability of obtaining 1, 3 or 5 is 1/6 plus 1/6 plus 1/6, equal to 3/6 or 1/2; in other words half the sides of the die have odd numbers, whereas the other half has even numbers, so we expect about half of the throws to yield an odd number.

Working with two dice adds challenge to our understanding. The most wanted number is a double six, but we intuitively know that the probabilities of getting it are somehow small. A common mistake is to add the probability of the first die yielding a six to the second die yielding a six, which would be 1/6 + 1/6 = 2/6 or 1/3; instead of adding, we have to multiply the probabilities of such an outcome; that is, obtaining the first six has a probability of 1/6, and obtaining the second six has the same, giving 1/6 x 1/6 = 1/36, that is, we can *expect* to see a double six in every set of about 36 throws in average (remember, probabilities are *not* exact). This can be understood (again intuitively) because it is rarer to get a double six than a single six. Of course, it goes the same for every double five, four, three etcetera.

A variant of two dice throwing is calculating the probabilities of getting a number as result of an addition, this is, 2, 3, 4 … 12. Two cases results are obvious, 2 and 12, for which the probability is of 1/36, as explained in the paragraph above. What about the other numbers?

If we want a 3, we can obtain it in two ways, die A giving a 1 and die B giving a 2, or vice versa, die A giving a 2 and die B giving a 1. The first combination has a probability of 1/36 and the second combination has the same probability. So we have two times 1/36 the probability of getting this result, which makes a probability of 2/36.

If we want a 7, the following combinations and probabilities are shown in table 1.

Combination	**Dice A**	**Dice B**	**Probability**
1	1	6	1/36
2	2	5	1/36
3	3	4	1/36
4	4	3	1/36
5	5	2	1/36
6	6	1	1/36
Sum of Probabilities			6/36 = 1/6

Table 1. Combinations of two dice whose addition yields seven, and their probabilities.

In other words, we can expect to get our seven around once every six throws.

It is obvious that we can calculate the probability of obtaining any number between 2 and 12 by this simple and easy to understand method, and it will be interesting to observe a pattern that will come to light if we do it systematically:

Two dice Throw result	**Number of possible permutations**	**Probability of a given result**
2	1	1/36
3	2	2/36 = 1/18
4	3	3/36 = 1/12
5	4	4/36 = 1/9
6	5	5/36
7	6	6/36 = 1/6
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36

Table 2: Number of permutations and their probabilities by throwing 2 dice.

As it can be seen, the more ways to obtain a certain number, the higher its probability to come out. This game can be performed using ten side dice, which makes necessary more arithmetical calculations, but certainly things will turn even more challenging with three (or more) regular dice.

Birthday Coincidence: Lesson Three Teacher's Background

Lesson 3 deals with something surprising for many people, even if it is not as rare as we perceive it, given the condition of having the necessary number of people. It is about coinciding with someone else in our birth day.

Let's start simple: if we divide an average month, we come out with the distribution of days into four weeks, I, II, III and IV as shown in table 3.

Week	Calendar Day
I	1 – 7
II	8 – 15
III	16 – 23
IV	24 – 31

Table 3. Distribution of a month's days into four weeks

If we take a single person, the probability is definitely zero; if we bring together five persons, certainly two of them must have been born the same week (regardless of month) because there are only four weeks per month. This is a one hundred percent probability, or 1, but how to know which are the probabilities of coincidence with two, three, and four persons? It seems reasonable that four persons will have a probability of around 75% of coincidence, three persons around 50% and finally two persons around 25%.

We can now extend the exercise using the twelve months of the year. In order to have a coincidence between two persons, we need a group of 13 people, equal to 12 + 1… why 13? This is because in the worst (?) of the cases, the twelve first individuals will not coincide in their month of birth, but the person number 13 must be born in the same month as someone else.

A probability of 50% arises with 6 + 1 persons, and consequently the probabilities of 25% and 75% occur with 3 + 1 and 9 + 1 persons respectively; each additional person increases the probability in approximately 8.33%, or 1/12.

We can enlarge the game to coincidences in the day of the month, from 1 to 31, or the day of the year, from 1 to 365 … you do the math!

In fact, the probability calculation of this birth date coincidences is more complex than presented here, but such statistics is not for elementary school level. The pedagogical importance of presenting here this simplified method of calculation is that it shows the logic behind the calculation, which gives the students the thinking skills for building a mathematical procedure in order to solve a probability problem.

Drawing a Ball from the Bag: Lesson Four Teacher's Background

Lesson four is about sampling with and without replacement by drawing balls from a bag. We have red (R) and green (G) balls numbered as follows R1, R2, G3, G4, G5. The probabilities of a ball being drawn from the pool are shown in table 4.

Draw	**Probability**
Any ball	20 %
Red	40 %
Green	60 %
Pair number	40 %
Odd number	60 %
Red pair	20 %
Red odd	20 %
Green pair	20 %
Green odd	40 %

Table 4. Probability of drawing a ball from the pool.

Of course these probabilities stand only for the first draw or if we replace the balls in the bag. On the other hand, if we do not replace the balls, the probabilities of the balls to be drawn changes as there are less balls in the bag until eventually the last ball will have a 100 % probability to be drawn.

Combining Three out of Five Flavors: Lesson Five teacher's Background

In lesson five the first contact with combinatorial analysis will be made. To the teacher, it is useful to know the formulas for this type of analysis. Combinatorial analysis deals with permutations and combinations (in this background, we will deal only with permutation and combination without repetition), and it makes use of factorial numbers, notated as n!

The difference between permutation and combination is that permutation takes into account the order in which the different elements in a selection appear, that is, AB and BA are two different permutations of two elements whereas in combination the order of appearance is not taken into account, that is, AB and BA are the same combination of these elements.

Factorial numbers are those which are multiplied by its preceding number up to one, for example, 3! = 3 X 2 X 1 = 6. Indeed, the number of permutations of n elements out of n is its factorial, shown in table 5, which presents the possible permutations of the letters A, B and C… but all of them are the same combination of letters.

Table 5. Permutations of three different elements, A, B and C.

	Permutation
1	A B C
2	A C B
3	B C A
4	B A C
5	C A B
6	C B A

But, what happens when we want to know the number of possible combinations (without replacement) of "r" number of objects drawn from "n" objects? Here is where we use the formula

$$C_r^n = \frac{n!}{(n-r)!(r!)}$$

where:

c is the possible number of combinations,

n is the total number of items (or objects),

r is the number of items selected from the total, and

! is the factorial of a number (by definition 0! = 1).

A good practice for the teacher is to calculate the number of different combinations out of six different elements, namely A, B, C, D, E and F.

Taking all six elements, there is only one possible combination: A, B, C, D, E, F. (the order of the arrangement is not taken into account). This can be calculated by using the formula above:

$$C_6^6 = \frac{6!}{(6-6)!\,(6!)} = \frac{6 X 5 X 4 X 3 X 2 X 1}{(0)!\,(6 X 5 X 4 X 3 X 2 X 1)} = \frac{720}{(1)\,720} = 1$$

Taking five elements at a time, we have the six combinations shown in table 6.

Table 6. Combinations of five elements out of six different elements, A, B, C, D, E and F.

	Combination				
1	A	B	C	D	E
2	A	B	C	D	F
3	A	B	C	E	F
4	A	B	D	E	F
5	A	C	D	E	F
6	B	C	D	E	F

Using the formula we calculate:

$$C_5^6 = \frac{6!}{(6-5)!\,(5!)} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{(1)!\,(5 \times 4 \times 3 \times 2 \times 1)} = \frac{720}{(1)\,120} = 6$$

Taking three elements at a time, we have:

$$C_3^6 = \frac{6!}{(6-3)!\,(3!)} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{(3)!\,(3 \times 2 \times 1)} = \frac{720}{(6)\,(6)} = \frac{720}{36} = 20$$

If we calculate the combinations from zero through six elements out of six elements we obtain the results shown in table 7.

Table 7. Combinations from zero through six elements out of six elements.

***r* out of *n* elements**	**Number of combinations**
C_0^6	1
C_1^6	6
C_2^6	15
C_3^6	20
C_4^6	15
C_5^6	6
C_6^6	1

* remember that 0! = 1

It is worth mentioning that as shown in Table 2, there is a symmetrical distribution of the number of combinations, which peaks at the central value, in this case three elements out of six.

Perhaps we do not want to go in so much detail during our lessons, in which only one example or two are studied. Formulas can be avoided, but the most important is to teach the students the logical thinking and the mechanics of sequencing when sorting out combinations of a limited number of elements out of the pool.

Rare Body Traits: Lesson Six Teacher's Background

In lesson six, the frequency of appearance of certain inherited traits is demonstrated. The following concepts are to be explained with this exercise:

> Frequency of occurrence: how often a given inherited trait appears in the population, in other words, in how many individuals from a sample it appears.
>
> Relative frequency is the fraction out of our entire population equal to one, or the percentage out of our entire population equal to hundred per cent, of the appearance of a given trait.

For this exercise it is better to choose inherited traits which are easily recognizable and in sight. A first choice trait is the earlobe shape, which can be detached or attached form the head side as shown in figure 1.

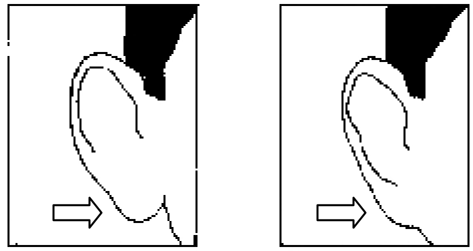


Figure1. Detached (left) and attached (right) ear lobe.

In general terms, the detached earlobe is more frequent than its attached counterpart, which can be demonstrated within the student population.

Another trait, less common than the attached earlobe is the “widow’s peak” in the head’s front hair, as opposed to the usual round head’s front hair, as shown in figure 2.

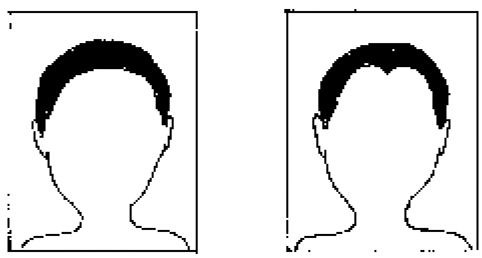


Figure 2. Round head’s front hair (left) and “widow’s peak” hair (right).

In order to know the frequency and relative frequency of occurrence of such traits we need to make the direct count in our population and use the formulas below:

$$\text{Relative frequency of a given trait} = \frac{\text{Number of occurrences}}{\text{Total number of individuals}}$$

$$\text{Percentage occurrence of a given trait} = \frac{\text{Number of occurrences X 100}}{\text{Total number of individuals}}$$

By working with the students themselves, you can find out the relative frequency of appearance of such (or other) traits. What about investigating the fraction (or the approximate number of individuals in a population) having both rare traits? As with the dice throwing from lesson two, a common mistake is to add up both fractions, which is a misleading idea. In order to calculate the percentage of individuals having both rare traits we must multiply both fractions, which obviously will yield an even smaller fraction than the initial separate fractions. This will be better illustrated in the suggested lesson.

Finally, it is worth mentioning that for this topic a good sample number (number of individuals) is thirty or more students, but in case your class is significantly smaller than this, a good idea is to merge two or more classes in order to get a sample of 30 or more students (that is what I did). This will give more realistic and reliable results about the fraction of a population having a rare trait, which can in turn be extrapolated to another sample of students or a larger population sample.

LESSON PLANS

While thinking these model lessons I realized that I had to either suggest a general and theoretical procedure for doing the activities or “invent” ideal examples to show. Instead, it seemed a better idea to do present the activities that I have done with my classes as a model of real situations. Of course, every time you do these activities, randomness can play its part in probability, making your results slightly different, but still reasonably close to the theoretical results; the exception could be combinatorial analysis in which the procedure exposed here does not get into randomization. So, plan your activities beforehand, lead your students through logical thinking, and enjoy the game!

Lesson One: Tossing the Coin

One coin was given for each student to toss and record the outcome as head (+) or tail (–). The assumption is that half of the times the coin is going to fall head-up and half of the times it is going to fall tail-up. The results for the first trial are given in table 8.

Table 8. Heads (+) and tails (-) from a set of 31 coin tosses (St. is the student number).

St.	Head (+) or tail (-)	St.	Head (+) or tail (-)	St.	Head (+) or tail (-)
1	+	14	+	27	+
2	+	15	-	28	-
3	-	16	+	29	-
4	+	17	+	30	+
5	-	18	+	31	+
6	+	19	-	32	-
7	-	20	-	33	-
8	+	21	+	34	+
9	-	22	-	35	+
10	-	23	+	36	+
11	+	24	-	37	-
12	+	25	-		
13	-	26	+	sum	+ = 20; - = 17

Two more trials were performed and the results are shown in table 9.

Table 9: Heads (+) and tails (-) from three trials (37 coin tosses each trial).

Trial	Heads	Tails
1	20	17
2	18	19
3	21	16
sum	59	52

The resulting percentages of heads and tails are as follows:

Heads: 59 = 53.1 %
Tails: 52 = 46.9 %
Total: 111 = 100.0 %

which is close to the theoretical result of 50 % heads and 50 % tails.

Lesson Two: Throwing the Dice

A “warm up” activity is to let the students throw a single dice for a sufficient number of times to realize that any side has the same probability to face up, that is, any number has 1/6 of probability to come out. This activity, if desired, can be also used as a lesson by itself. The results of a single throw per student are shown in table 9.

Table 9. Results from a single dice throw by 37 students.

St.	Number Facing Up	St.	Number Facing Up	St.	Number Facing up
1	3	14	2	27	3
2	5	15	5	28	5
3	3	16	1	29	6
4	4	17	4	30	4
5	6	18	2	31	1
6	1	19	6	32	4
7	2	20	3	33	1
8	6	21	4	34	1
9	2	22	6	35	5
10	4	23	5	36	3
11	3	24	6	37	4
12	6	25	4		
13	6	26	2		

Rearranging the results shows us that each number came out around six times, in other words, roughly 1/6 of the 37 throws, as presented in table 10.

Table 10. Number of occurrences for each number out of 37 throws.

Number	1	2	3	4	5	6
Occurrences	5	5	6	8	5	8

The two dice throw activity consists of calculating and obtaining the probability of getting the "lucky throw" which is a double six. As previously explained in the teacher background section, a six has a probability of 1/6 to face up, and a second six has as well a probability of 1/6. Obtaining two sixes in a double throw is more difficult than not obtaining them, and the probability is 1/6 X 1/6 = 1/36.

For the experimental part of the lesson I asked every student to throw two dice, which is considered an event, and record their results, which are shown in table 11.

Table 11. Results of 37 throws of two dice. For this exercise, the arrangement of the two faces up is indistinct, that is, 2, 4 is the same as 4, 2.

St.	Two dice faces up	St.	Two dice faces up	St.	Two dice faces up
1	1, 2	14	2, 5	27	2, 5
2	1, 6	15	1, 2	28	2, 6
3	2, 3	16	5, 5	29	1, 4
4	2, 6	17	1, 6	30	2, 6
5	2, 5	18	3, 6	31	3, 6
6	3, 5	19	5, 6	32	4, 5
7	4, 6	20	2, 4	33	1, 6
8	2, 5	21	5, 6	34	2, 6
9	5, 6	22	2, 6	35	4, 6
10	4, 5	23	1, 6	36	5, 6
11	5, 6	24	2, 5	37	2, 5
12	6, 6	25	4, 6		
13	2, 4	26	5, 6		

Only one out of 37 events was a double six, which is in agreement with the theoretical calculation of probabilities equal to 1/36. I must say that a double six could have aroused two times or none (as it was for a double one, which never came out), in which case the whole experiment should have been repeated one or two times more, which brings the experiment closer to the theoretical calculations, because the larger the sample, the more the obtained results are similar to the expected ones.

Rearranging the information from Table 11, the frequency of appearance of each possible two dice throw outcome is shown in Table 12.

Table 12. Frequency of appearance of each possible two dice throw outcome. Both dice numbers are added, that is, 1, 1 = 2.

Outcome	2	3	4	5	6	7	8	9	10	11	12
Frequency		2/37	2/37	2/37	2/37	10/37	7/37	4/37	4/37	6/37	1/37

It can be observed that, as discussed in the teacher background, the most common number is seven because it has more ways to come out (1, 6; 2, 5; 3, 4), and the frequency decreases towards the extreme values of one and twelve; the high occurrence of eleven can be attributed to random effect.

Lesson Three: Birthday Coincidence

For this lesson the students were asked their birth date, being the day (not the month) the relevant information. The days of the month, from 1 through 31 were grouped into four weeks (Roman numerals) as follows I: 1-7; II: 8-15; III: 16-23 and IV: 24-31. The data are presented below in table 13.

Table 13. Day of the month and week (Roman numeral) of birth of thirty seven students.

St.	Date and week	St.	Date of birth	St.	Date of birth
1	9: II	14	28: IV	27	14: II
2	24: IV	15	5: I	28	6: I
3	31: IV	16	18: III	29	8: II
4	1: I	17	3: I	30	22: III
5	26: IV	18	27: IV	31	29: IV
6	10: II	19	10: II	32	28: IV
7	16: III	20	20: III	33	1: I
8	4: I	21	12: II	34	8: II
9	30: IV	22	21: III	35	18: III
10	17: III	23	12: II	36	23: III
11	4: I	24	6: I	37	13: II
12	11: II	25	14: II		
13	18: III	26	9: II		

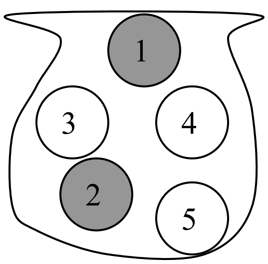
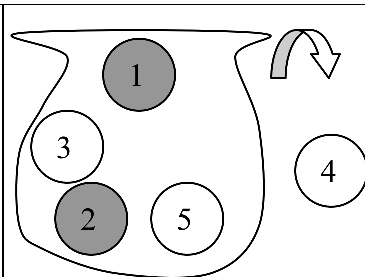
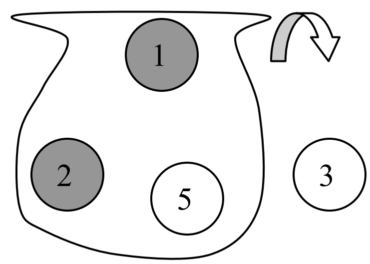
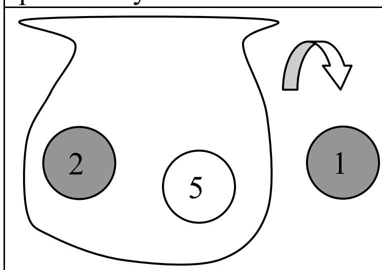
The analysis of the table, accepting that the arrangement by list number is indeed randomized, reveals that if we take pairs of students, there are not many birth week coincidences; if we take three students at a time there are more coincidences; when we take four students at a time there are much more coincidences, but obviously, if we take five students at a time, regardless where we start in the list, there is always a coincidence, this is, a 100% probability of finding at least two students born the same week.

Lesson Four: Drawing a Ball from the Bag

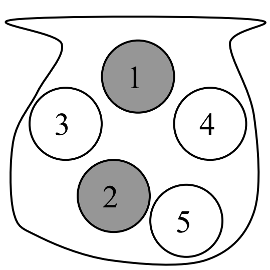
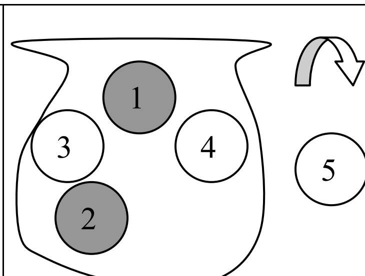
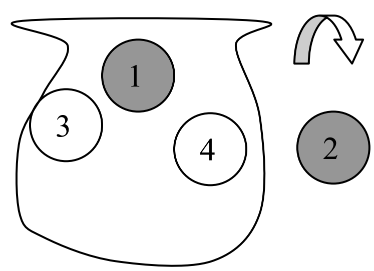
For this lesson the students were grouped in teams of five and each team received a bag containing three red (R) and two green (G) marbles numbered as follows R1, R2, G3, G4, G5. Each team had to choose a particular ball and calculate the probabilities for it to come out at each draw; these probabilities had to be calculated in function of three factors, namely

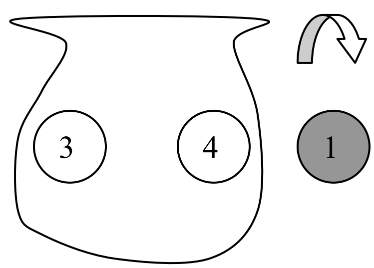
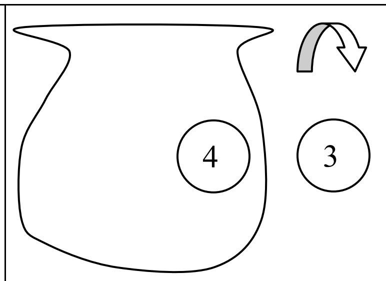
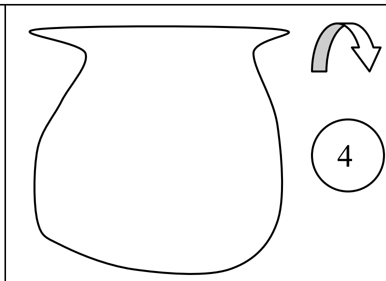
Color: red (R) or green (G).
Number: odd (O) or pair (P).
Particular ball: for instance R1.

Team one chose ball red one R1, and the probabilities for it to come out were calculated for each draw:

		
Start, before first draw probability of red: 40 % probability of odd nr: 60 % probability of R1: 20 %	1st draw: G4, after which: probability of red: 5% probability of odd nr: 50 % probability of R1: 25 %	2nd draw: G3, after which probability of red: 66 % probability of odd nr: 66 % probability of R1: 33 %
		
3rd draw: R1, after which probability of red: 50 % probability of odd nr: 50 % probability of R1: 0 %		

Team four chose ball green four G4, and the probabilities for it to come out were calculated for each draw:

		
Start, before first draw probability of green: 60 % probability of pair: 40 % probability of G4: 20 %	1st draw: G5, after which: probability of green: 50 % probability of pair: 50 % probability of G4: 25 %	2nd draw: R2, after which: probability of green: 66 % probability of pair: 33 % probability of G4: 33 %

		
3rd draw: R1, after which: probability of green: 100 % probability of pair: 50 % probability of G4: 50 %	4th draw: G3, after which: probability of green: 100 % probability of pair: 100 % probability of G4: 100 %	5th draw: G4.

After these exercises it is easy to understand that the probabilities of drawing a color, number, or specific ball change with every draw we make because of the balls remaining in the bag.

Lesson Five: Combining Three out of Five Flavors

Imagine we have an ice cream business and we sell five different flavors, namely apple (A), berry (B), citrus (C), date (D), and E vitamin (E). Our best client always asks for a three scoop serving, and he wants to have as many different combinations of three different flavors as possible. How can we figure out how many combinations of three different flavors out of five are possible? Remember that as a combination, ABC is the same as CBA.

On the one hand, we could start just putting flavors together and forming combinations as they come to our mind, but in that way it will take a long time to solve the question; moreover, we risk to repeat combinations by writing the same flavors in different order.

The best way to find out all the possible combinations of three elements out of five is to follow a sequence of choices, starting from the most obvious three flavors combination, as shown in table 14.

Table 14. Combinations of three out of five different flavors (ABC = CBA).

Flavor	1	2	3	4	5	6	7	8	9	10
Apple	A	A	A	A	A	A				
Berry	B	B	B				B	B	B	
Citrus	C			C	C		C	C		C
Date		D		D		D	D		D	D
E-vitamin			E		E	E		E	E	E

In studying probabilities, any flavor, let's say A, has 6/10 chances to appear, the combination of two specific flavors, let's say A B, has 3/10 chances to appear and any particular combination of three flavors, let's say ABC, has 1/10 probabilities to come out.

Lesson Six: Rare Body Traits

In this lesson we studied the frequency of occurrence of two body traits, the earlobe, detached/attached and the frontal head hair, round/widow's peak (refer to the background for the teacher) The population sample was the class of 37 students, the individual traits are shown in Table 15.

Table 15. Occurrence of detached (D) or attached (A) earlobe and round (R) or widow's peak (W) front head hair in a sample of 37 students.

St.	Ear lobe, Front hair	St.	Ear lobe, Front hair	St.	Ear lobe, Front hair
1	A,R	14	D,R	27	D,R
2	D,R	15	A,R	28	D,R
3	A,R	16	D,R	29	D,R
4	A,R	17	D,R	30	A,R
5	D,R	18	A,R	31	D,W
6	D,R	19	A,R	32	D,R
7	A,R	20	D,R	33	A,R
8	D,R	21	D,R	34	D,R
9	D,R	22	D,R	35	A,R
10	D,W	23	D,R	36	D,R
11	D,R	24	A,R	37	D,R
12	D,R	25	D,R		
13	A,R	26	D,R		

According to the table, 25/37 students have detached earlobe, this is about two thirds or a fraction of 0.68 (68%) from the total; in consequence, the fraction of students having attached earlobe is about one third, or 0.32 (32%) from the whole sample.

Regarding the front hair, the commonest is the round hair, which represents 0.94 (94%) from the total; in consequence 0.06 (6%) of the students have widow's peak hair.

If we want to make an estimate of the probability of appearance of two of these traits combined, let us not be mislead by adding the fractions corresponding to such traits; rather, we must multiply them (refer to Throwing the Dice background for the teacher). If we want to estimate the percentage of people who would have attached earlobe and round front hair, assuming these traits are independent, it would be

A,R = 0.32 X 0.94 = 0.30 or 30%

The actual count of "A,R students" is 12, which is 32 % of the sample.

In the case of students with attached earlobe and widow's peak (the two rare traits together) the calculation is

A,W = 0.32 X 0.06 = 0.02 or 2 %

A 2 % appearance of these two traits means that we could expect 1 AW in every 50 students, and as our sample is smaller than fifty, it was unlikely to have even one AW.

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