Probability of Events

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INTRODUCTION

We know understanding math and being comfortable with numbers is an important skill. Even so, many adults are afraid of math and do not hesitate to openly state that they are not good at math. Politicians and educators are concerned about the condition of education in our nation for various reasons (Brooks 3). Schools, administrators and teachers are under constant pressure to improve their students' math scores. High school teachers and college professors often complain that their students lack motivation and are disengaged from their own learning ("Teaching College Freshman").

There is a tendency amongst teenagers to be impulsive and risk seeking. They are enticed by short-term benefits. Current research suggests that risk-taking behavior may be attributed to the adolescents' still developing brains ("Brain Differences in Adolescents"). Teachers can benefit from this knowledge and help provide their students with small gains. The Algebra Benchmark Program at Lee enables students to accomplish small achievable goals.

ALGEBRA BENCHMARK PROGRAM AT LEE HIGH SCHOOL

I teach math at Lee High School in the Houston Independent School District. Lee High School serves a large geographic area bordered by the communities of Bellaire, Alief, Katy, and Spring Branch. ("Lee High School"). Lee is a Title 1 school and over 95% of our students qualify for free lunch. Many of our students are recent immigrants and have limited English language skills ("School Profiles").

Students generally enter a math program with little or no confidence. My students completed an information sheet at the beginning of the academic year, and over 50% of my students openly admitted that they did not like math. Students at our school are enrolled in an algebra benchmark program. The goal of the benchmark program is to help boost students' confidence by enabling them to experience success with a small amount of math material. It also provides them with multiple opportunities to show mastery of the material they learn.

The entire Algebra I is a year-long program as opposed to two semesters of Algebra IA and Algebra IB. Our classes are double blocked and students have Algebra 1 every single day. Our course is divided into 61 benchmarks. Each benchmark represents a simplified math objective and is grouped in units consisting of 4 or more benchmarks. Students have to master 80% of the benchmarks to get course credit for Algebra I. Thus, students need to master a total of 48 out of 61 benchmarks.

The benchmark program ensures students continue to work towards mastery until the last week of school. In the event that a student is unable to master 80%, say they are short of some benchmarks, they can get the balance number of benchmarks during summer school. Our students get to keep the benchmarks they earned during the academic year. Upon completion of 80% of the benchmarks, students graduate from the program and can choose to stop coming to summer school. Thus they do not have to remain enrolled for the entire duration of the summer school. This is another incentive for students to work hard and be successful at their own pace.

Each benchmark is assessed using four problems. Students have to correctly answer three problems out of four to show that they have mastered a benchmark. Two problems from a benchmark are on a unit test. To assess four problems, each benchmark appears on two unit tests. Each benchmark gets tested at least twice. There are mini-benchmark make-up tests for each benchmark that are also incorporated into the cycle of teaching and re-teaching.

Upon mastery of a benchmark, the student is recognized in the presence of her/his peers. S/he is presented with a smiley face sticker that s/he places on a benchmark progress chart posted in the classroom. Mastery of every 10 benchmarks marks another occasion for students to receive a lapel pin. The pin shows the number of benchmarks they have mastered. Students at our school need to wear their identity cards. Our students pin their benchmark pins on the identity card cord. Some pin it to their clothes or bag. Rewarding our students with benchmark pins, is another way for students to publicly advertise their mastery of Algebra. It helps build math confidence.

Last but not least, during the year, students who have mastered a specific number of benchmarks at each quarter are invited to enjoy an ice-cream sundae during school lunch. Students get a personal invitation in presence of their peers. Students love the attention and look forward to the sundae events. This is another way to keep the students motivated and engaged in learning and monitoring their own progress. We also gave away T-shirts to students who had mastered the required number of benchmarks for the first semester. The T-shirt design was chosen from a host of designs submitted by our students. Math teachers at Lee constantly look for ways to motivate our students.

For many students success in school has little to do with true understanding and much to do with covering the curriculum in a given amount of time (Brooks 7). At our school, students can master benchmarks at their own pace. Students have an entire year to master the benchmarks. There are many opportunities for student to either learn from the teacher or get help from their peers during class and tutorials. Many teachers offer before school, during lunch and after school tutorials. We make transportation available to students who stay for after school tutorials. In addition, our school offers tutorials on Saturday mornings in the school library. Our school employs student-tutors to help tutor students one on one every Saturday mornings. This is another incentive for students to be more successful at math during the year. We also have volunteer-tutors from the Indian Institute of Technology. At least 25 students come each Saturday for tutoring. The tutorial attendance spikes and reaches to 50 or more students around report card time. Students get to eat a donut and practice material with the tutors before taking the makeup benchmark test.

As a Math teacher at Lee High School, my task is two fold. I have the challenge of changing my students' perceptions of math and teaching them the math skills they need to be successful. In line with my school's initiatives, I'd like my students to believe math is cool and doable. The benchmark program certainly helps and provides multiple opportunities to show mastery of benchmarks. The public display and recognition helps students feel successful and acquire confidence.

Having worked with the benchmark program over the past year, I feel confident about the program's goals. Students come during lunch to take their benchmark tests. Many are eager to show mastery of a benchmark and want to take the benchmark test immediately after they have been taught a benchmark. They look forward to taking mini benchmark make-up tests. We have benchmark marathon days in my class. Students get 20 minutes of study time to learn a benchmark of their choosing. They seek out students who have mastered the benchmark they need. Then we all test. Thus students teach and learn from each other.

UNDERSTANDING PROBABILITY AND STATISTICS IN EVERYDAY LIFE

The seminar at Houston Teachers' Institute is called Probability and Statistics in Everyday Life. Probabilistic ideas and statistical reasoning relate to gambling, lottery and other games of chance. It also relates to coincidences.

Birthday Phenomena

Coincidences and chances occur in our everyday life and amaze us. We wonder about them. Little do we know that mathematically, assuming independence of birthdays, there is approximately a 50% chance that two people amongst a group of 23 will have the same birthday. The 50% chances can be increased to 95% by increasing the number of people in the group from 23 to 48. There is a 95% chance that two people amongst a group of 48 people will have matching birthdays.

Heuristic Judgments and Biases

Most people make decisions based on belief or likelihood of certain events. Kahneman and Tversky state that people assess the probability of uncertain events by reducing the complex task of assessing probability to simpler judgmental operation called heuristic principles. According to the authors, these principles, although being useful, lead to severe and systematic errors (3). For example, in determining the distance or size of an object, people rely heavily on clarity and sharpness with which the object can be perceived. Thus they will either underestimate when visibility is good or overestimate when visibility is poor.

Also, people use subjective probabilities as opposed to objective probabilities. Probabilities, such as .0001 or .0002, seem small but are not insignificant and not equal to zero. The human mind simply equates small probabilities as essentially zero, and then people are surprised when these probabilities manifest as events. Misconceptions are not limited to laymen. Many researchers rely heavily on small sample size. The inherent belief in the representative nature of a sample irrespective of its size results in overestimation and over interpretation of findings (Kahneman and Tversky 8).

Rare Events and Variation

Law of large numbers states that with a large enough sample, any outrageous event is likely to happen. Rare events that happen only once in one million can happen many times when the population is increased from one million to 250 million. Mathematically, if a coincidence occurs to one person in one million each day, then we should expect 250 occurrences a day and close to 1000,000 such occurrences a year (Diaconis and Mosteller 859).

When comparing numbers from larger institution versus smaller institution, people fail to appreciate the fact that the smaller institution will show more variability. As an example Kahneman and Tversky compare births at a large hospital versus birth at a small hospital. For one year, each hospital tracked the number of days on which more that 60 percent of the babies born were boys. When asked which hospital recorded more such days, most people judged the chances to be the same for the large and the small hospital. According to sampling theory, the smaller hospital is likely to show more variation. A large hospital is likely to maintain the 50 percent rate boys and girls. "This fundamental notion of statistics is evidently not part of people's repertoire of intuitions" (Kahneman and Tversky 6).

Texas State Lotto Pick 3

State lotteries help generate revenue for a number of states. Lotto games involve selection of numbers. Many aspects of lottery are statistically relevant and can be explored as part of school coursework. Pick 3 is a Texas state lotto. Pick 3 is played twice a day. Each play cost 50 cents to play. There are twelve drawings in a week with a top prize of $500 on a $1 play. The game is easy to play and only needs 3 numbers to be picked from “0” to “9.” The player can pick the numbers or check quick pick box. Next the player can decide to play the numbers in exact, any or combo order.

The chance of being a Pick 3 winner is 1 in 1,000. There are 10 ways to pick the first number, second, and the third number. Numbers picked can be repeated. Thus there are a total of 10 x 10 x 10 = 1,000 ways of picking 3 numbers. The probability of winning the number in exact order are 1 in 1, 000. The payout is $250 for $1 play. The expected gain is the product of the payout times the probability 1/1,000 x $250 = .25. The expected gain is 25 cents. The odds of winning get better when playing Pick 3 in any order. The payout is $160 for a $1 play. The expected gain is slightly less than 48 cents, better than 25 cents. Expected gain is 1/333 x $160 = $.48.

Many people continue to play the lotto not knowing the expected values of the advertised payouts. The knowledge about expected value can help lotto players make informed choices.

Risky Choices and Expected Gains

The idea of expected gain is hard to fathom and shows how the human mind can be both risk averse and risk seeking at the same time. Kahneman and Tversky researched people’s attitudes towards risks. They noticed that attitudes varied and were contingent on how the problem was framed (456). When a problem is framed in terms of gains, humans show tendencies that are described as risk-averse tendencies. When the problem is framed in terms of losses, the same individual displays tendencies that are described as risk-seeking.

Statistical research is bound with many similar examples. A small informal experiment with a group of students revealed the risk-averse and risk-seeking tendencies of people. Present the following two situations, and ask people to choose from A and B in Situation 1 and Situation 2 separately:

Situation 1: Gain

- Choice A: Receive $1,000 with certainty
- Choice B: Receive $2,500 with 50 % probability

A clear majority of people will favor choice A. The expected gain for choice A is $1000. The expected gain for choice B is $1,250. The expected gain for the uncertain choice B is higher, yet respondents choose a lower expected gain by favoring choice A. They thus display risk-averse tendency.

Situation 2: Loss

- Choice A: Loose $1000 with certainty
- Choice B: Loose $2,500 with 50 % probability

For situation 2, many respondents will chose choice B. The expected loss for choice A is -$1000. The expected loss for choice B is -$1,250. The expected loss for choice A is lower, yet respondents choose a higher expected loss by favoring the uncertain choice B. They thus display risk-seeking tendencies.

Conditional Probability-Monty Hall Problem

Understanding conditional probability is a challenge. It is important to the understanding of statistics. For example probability of speaking English, given that s/he is American may be much higher than say the conditional probability that someone who speaks English is American. Monty Hall and the Prisoners Paradox are examples of conditional probability.

The name of the host of an American game show called *Let’s Make a Deal* is Monty Hall. He shows players three closed doors. There is a car behind one door and a goat behind the other two doors. The player will win the car if s/he correctly chooses the door with the car. S/he wins nothing when the door with the goat is chosen. The player selects a door. Before opening the player's door, the host opens one of the other two doors to reveal a goat. Monty Hall then asks the player if s/he would like to change her/his choice of door with the unopened door. The relevant question that factors in conditional probability is: Will the player increase her/his chances of winning by switching doors?

For a solution to the above problem let's look at the possible scenarios:

Scene	Door 1-Players Door	Door 2	Door 3
A	Car	Goat	Goat
B	Goat	Car	Goat
C	Goat	Goat	Car

Lets look at the probability of each scenario.

Scene	Door 1-Original Pick	Door 2	Door 3
A	Car	Goat *(revealed)*	Goat ***(swap& loose)***
B	Goat	Car ***(swap & win)***	Goat *(revealed)*
C	Goat	Goat *(revealed)*	Car ***(swap& win)***

In scene 1 if the player swaps, he is sure to lose. In scene B if the player swaps Door 1 with Door 2, he is sure to win. In scene C if the player swaps Door 1 with Door 3, he is sure to win. The player can win both in scene B and scene C where s/he swaps her/his choice of door with the other unopened door. Thus there is a two in three chance of winning once a door is opened to reveal a goat. The player can only lose in scene A. There is only one in three chance of losing. The game show host has provided the player with new information. If the player understands how the odds have changed, s/he stands to win. The player will increase her/his chances of winning by switching doors.

The solution to the Monty Hall problem is counter-intuitive. It is hard to believe that something has changed by revealing the goat behind one of the doors. The Prisoner's Paradox is another version of the Monty Hall problem. There are three prisoners. One of the three prisoners will be freed while the other two will be executed in the morning. One of the prisoners asks the guard to tell him which of the other two will go free. The prisoner argues and insists with the guard that this does not tell her/him about her/his own chances of being executed. The guard states that upon receiving the requested information, prisoner's chances of execution will increase.

Misconceptions about Independent Events

People believe that a random process will self-correct or balance itself not only in the long run but also for small trials. For example when flipping a coin, people would expect the coin to be fair and demonstrate its fairness when flipped multiple times. Consider the following results when a coin is flipped five times: H for heads and T for tails: H, T, T, H, T. Before flipping the coin a sixth time, many people would erroneously expect the next flip to result in a head in order to balance the results with three heads and three tails. Thus the belief leads them to perceive a higher likelihood of head versus tail on the sixth trial. In the long run the number of heads and tails will balance but not necessarily on the sixth trial. The coin does not remember what it flipped on its prior trial. The occurrence of head and tail are equally likely and does not depend on prior outcomes.

This misconception of self-correcting events is also known as the gambler's fallacy. After a long run of black on the roulette wheel, a gambler expects red to help balance the sequence of blacks. The idea that the past outcome influences the upcoming outcome is known as the gambler's fallacy. "Chance is commonly viewed as a self-correcting process in which deviation in one direction induces a deviation in the opposite direction to restore the equilibrium" (Kahneman and Tversky 7).

PRIOR KNOWLEDGE OF MY STUDENTS

Given our present setting that stresses performance, many students learn that technique, rules and memory matter more than context, authenticity and wholeness (Brooks 9). I'd like my students to seek deep understanding in addition to learning the short-term strategies that help them pass their benchmark tests. Our Algebra students meet every day for a total of 90 minutes except on Mondays when they meet for 80 minutes. I'd like my curriculum unit to be packed with interesting activities that will help students learn and not just perform on the benchmark tests. The curriculum unit will explore the idea of probability and predictions based on theoretical and experimental probability. The course Probability and Statistic in Everyday Life at the Houston Teacher Institute relates closely to at least five units in Algebra and one unit in Geometry.

Algebra Unit

- Data Analysis Unit
- Informal Patterns Unit
- Formal Patterns Unit
- Probability of Combinations of Events
- Predictions Based on Theoretical and Experimental Probability

Geometry Unit

- Linear Equations

The Data Analysis Unit closely relates to the unit on Probability of Combination of Events Unit and the Predictions Based on Theoretical and Experimental Probability. In the Data Analysis Unit students have to be able to interpret and create histograms, bar graphs and pie charts. They have to know how to calculate mean, median, mode and range of a set of data. Finally, they need to understand and be able to select an appropriate measure of central tendency for a set of data.

In Texas students are introduced to the idea of probability in third grade. In sixth grade students are exposed to measures of central tendency by learning how to calculate statistics such as the mean, median, and mode. Students also learn to calculate the range of a data set. They continue to explore the idea of probability and statistics in seventh and eighth grade. Thus students in ninth grade should have some idea of probability and be familiar with statistical measures, such as mean, median, mode, and range. They should be able to calculate the statistics.

My students can calculate the mean, median, mode, and range. They just need to be reminded about the procedure for calculating them. For a data set with 10 numbers: 6, 5, 3, 2, 1, 1, 5, 7, 5, 0. The mean is 3.5, median is 4, mode is 5, and the range is 7.

To calculate the mean find the sum of all the numbers in the data set.

Sum = 6+5+3+2+1+1+5+7+5+0 = 35
Mean = Sum divided by 10
Mean = $\frac{35}{10}$
Mean = 3.5

To calculate the median, for the same data set as above, all the numbers need to be arranged in ascending order. It is number in the middle position or average of two numbers in the middle positions. The median divides the data into lower half and upper half.

0, 1, 1, 2, 3, 5, 5, 5, 6, 7.
Median = $\frac{(3+5)}{2}$
Median = 4

The mode is the number that occurs the most.

0, 1, 1, 2, 3, 5, 5, 5, 6, 7
Mode = 5

The range is the difference between the maximum and minimum.

Range = 7-0
Range = 7

Generally my students do well with creating and making graphs. However, the hardest thing for my students to do is to interpret graphs and master the selection of an appropriate measure of central tendency. I think the reason for this is that interpretation requires a higher order skill. It simply cannot be learned with the help of a technique or short cut. It requires deeper understanding prompted by thoughtful questions. The student has to look beyond the apparent, delve into issues deeply, and form their own understanding of the situation (Brooks 110). The unit aims to ask thoughtful questions during the lessons.

OVERVIEW OF THE CURRICULUM UNIT

Traditionally math is taught as a body of information and procedures. Understanding needs to come to the forefront and not take a back seat. There is wide agreement among researchers that a constructive, active learning approach must be used when teaching mathematics (Burns and McLaughlin 1). Hence in the unit on Probability, the activities will be structured to allow student-to-student interaction in cooperative settings. Brooks states that “cooperative learning experiences have prompted interpersonal attraction among initially prejudiced peers and such experiences have prompted interethnic interaction in both instructional and free-time activities” (3). Also, when student-to-student interaction is encouraged, students begin the experience the richness of the topic taught and begins to ask and seek answers to their own questions.

The unit will be comprised of at least four 90-minute block periods. Students will experiment with flipping coins and rolling dice. The following concepts and skills will be developed during the course of this unit:

- Probability is a number between zero and one
- Equally likely events
- Independent events
- Idea of “long run”
- Theoretical probability
- Observed probability based on experimentation
- Calculation of probabilities of equally likely events

The unit will be interactive. Students will participate in activities adapted from the statistic unit of the Interactive Mathematic Program (IMP) published by Key Curriculum Press. IMP is a grown collaboration of mathematicians, teacher-educators, and teachers. IMP has created a four-year program of problem-based mathematics that replaces the traditional Algebra I-Geometry-Algebra II/Trignometry-Precalculus sequence (Fendel vii).

Probabilistic thinking is counter-intuitive and it is important for the activities to be concrete. Students persistently believe in luck and it often takes a great deal of experience before they become comfortable with the probabilistic notion (Fendel xxiv). The unit will also allow students to explore and recognize the gambler's fallacy (that the previous result of a roll will influence the outcome of next roll). Basic concepts of probability, including independent events, observed versus theoretical probability will be introduced. Students will continue to work on different problems to develop concepts and skills needed to master the unit on Probability.

When exploring results of observed probability based on performing experiments and theoretical probability, the class will gather data and build graphs. Students will use this graph and write a summary of what they think the graph is communicating. The class will continue to explore and think of different ways the same information can be presented. Can this graph be presented differently? Can we create a pie chart? When should a bar graph be used? When should a pie chart be used? Is a bar graph better than a pie chart?

How can mean and median be determined from bar graphs? Students will realize the difference between numerical data and categorical data. They will learn to calculate mean and median of numerical data. To review the material covered in the Data Analysis Unit and keep the pace moving, during the unit on Probability, we will use the graphing calculator's statistic menu to calculate mean and median. We will use the computer lab to do some guided research on how newspaper articles use these statistics and what they communicate to us.

At the end of this unit, students will design their own game using the idea of probability. They will write the rules to play the game. They can choose to work in pairs to create their game. The students will make presentations and display their game in a gallery. Students can be invited to play some of the displayed games and vote for the best game.

LESSON PLANS

Lesson One: The Meaning of Strategy *90 Minutes*

This lesson will introduce the idea of strategy and long run by letting students play a game. It will enable the participants to cooperate, make conjectures and test them through experimentation and data collection. The game revolves around the idea of expected value. It will not be explored directly in our lesson. Hopefully it will spark the students' interest and attempt to answer what helps determine a good strategy.

Material

Paper, pencil, flip chart/white board, calculator, markers, and dice

Activity One (Large Group)

Ensure that students understand the meaning of the word "strategy." What is the meaning of the word strategy? Do you have strategies for completing your chores at home? What are some strategies you use to make your parents buy you new clothes and shoes? Do they have a strategy to study for passing the TAKS? The students will use a six-sided die, labeled 1 through 6 to play a game adapted from the IMP unit.

The students will work in groups of four. Each group will play several turns. Each turn consists of one or more rolls. Each student keeps rolling until s/he chooses to stop or rolls a 6. If a student stops rolling before rolling a 6, his or her score for that turn is the sum of all the numbers rolled. Example: A student rolls a 3, 4, and 5 and decides to stop. His score for this turn is 3+4+5=12. If the student rolls a 6, then her/his score for that turn is 0. Example: Another student rolls a 3, 4, 5, and 6. Her/his turn is over since s/he rolled a 6, and her/his score is 0.

Activity Two (Small Group)

Allow each student in the group to play the game for 10 turns the first time. Help students distinguish between a turn and a roll. As the students play the game, walk around the room and encourage students to talk about the strategies they are using. They need to decide when to stop or continue rolling. Each student can experiment with several strategies at first. As the students are experimenting, each group creates a list of the strategies it used. Create a master list of strategies and invite students to write down their strategies on a board/flip chart.

Name of Student	Description of Strategy	Points Scored

As a whole class review some of the strategies and check if the strategies are complete or not. A strategy clearly needs to state under what condition the player will stop rolling. For example, roll at least three times is not a complete strategy. It does not tell the player when s/he should stop rolling. A strategy can be to roll if your score is less than 10. This strategy tells the player that s/he needs to stop once her/his score is 10 or more.

Activity Three (Small Group)

Ask each group to pick a strategy. Play the game using the strategy picked for 10 turns. Let the students report their scores and the strategy they used on a chart.

Group of Student	Description of Strategy	Points Scored

As a whole class, after the results have been compiled, review the results. Point out some high and low scores. Invite the class to respond to some thoughtful questions. It will help students articulate the idea of strategy with regard to the game. How do you decide which of these strategies are good strategies? For example, if two players are playing the game, and player A has played 10 turns and made 50 points, while player B played 8 turns and made 50 points, which player seemed to have used a better strategy? Here is an opportunity to help focus our students' attention to the idea of average points per turn and draw their attention to expected value.

Activity Four (Large Group)

Lastly, invite the class to choose a strategy from the list of strategies compiled. The class can then re-play the game for 10 more turns. A student can volunteer or the teacher can choose a student to play one turn. The first student will choose a student to play the second turn and so on. As the class is playing the game, each student should track the class score. This will keep the class engaged while individual students are taking turns. Check with the class and decide how this score compared with previous scores. Was this indeed was a good strategy? Why? Why not? Use the data compiled to review the procedure to calculate mean, median, mode, and range of the scores.

Extension

To extend this activity, ask the students to start designing their own game. What rules would their game follow? Finally, invite the class to play the game they created with their parents or siblings. The students must record the scores. They must also find the mean, median, mode, and range of each game.

Lesson 2: How Many Rolls for a Two *90 Minutes*

In this lesson students will collect data. The lesson provides a good opportunity to introduce histogram, bar graph, and review mean, median, mode, and range.

Materials

Paper, pencil, rulers, color pencils, graph paper, large graph paper, calculators, flip chart/white board, markers, dice and pennies

Activity One (Individual Work)

Students roll a dice until they get a 2. The students continue to roll and record the number of rolls it takes to get a 2. Model the activity with an example: First roll and say it results in a 3, second roll results in a 6, third roll results in a 1, and fourth roll results in a 2. Thus it took 4 rolls to get a 2. Ask the students to predict the average number of rolls it will take to get a 2. Each student can do the experiment 10 times and keep a record of the number of rolls.

Ask the students to find the range of the number of rolls it took to get a 2. What is the largest number of rolls it took to get a 2? What is the smallest number of rolls to get a 2? What is the average number of rolls for each of the ten experiments? How close is it to your prediction? If you had to revise your prediction, what would it be and how do you determine it? Some students in the class may see that you get a 2 one-sixth of the time. $P(2) = 1/6$. On an average it takes about six rolls to get any of the numbers 1, 2, 3, 4, 5, or 6 on a six-sided dice.
$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$.

Activity Two (Large Group)

Work as a class to get a class average. Make a master list with students' names and have students write down the number of rolls for each turn. Use the table to find the class average.

Student Name	Total Number of Rolls Needed

Create another master list to make a frequency histogram.

Number of Rolls	Number of Times this Occurred

Activity Three (Individual Work)

Students need to represent the data in different graphs. They can choose to use a histogram, bar graph, or pie chart to present the data. Students must justify and explain the reasons for choosing a particular kind of graph as a way of representation.

Activity Four (Large Group)

List the sample space for the rolling a die: {1, 2, 3, 4, 5, 6} Find probability of rolling a 0, 1, 2, 3, 4, 5, 6, or 7. Write the probabilities as fractions, reduced fractions, decimals, and percents.

Extension

To explore likely verses not likely, have the students flip a coin 50 times. It is unlikely that the experiment will result in 25 heads and 25 tails. It is more likely that the number of heads (or tails) was less than the number of tails (or heads). Examine with the class how many students got exactly 25 heads and 25 tails. How many students got more heads compared to tails? How many students got more tails? Compile a class data set and get total number of heads and tails.

Provide students with TAKS questions and have them practice finding mean, median, mode, and range of data sets. Teach the students about the Stat function on the calculator. Show the students how to read the 1-variable statistics results. It calculates the mean and median. It also shows the minimum and the maximum that helps calculate a range for a set of data points.

Lesson Three: Independent Event *90 Minutes*

This lesson will help explore the idea of independent events. Both children and adults do not comprehend the meaning of independent events. We fail to see that flipping a coin, rolling a die, and spinning a roulette wheel are all independent events.

Materials

Paper, pencil, rulers, color pencils, graph paper, large graph paper, calculators, flip chart/white board, markers and pennies

Activity One (Large Group)

Flip a coin five times, and note the outcome. For example let's say the outcomes are in the following order TTHTT. (T for tail and H for head). Before flipping again, ask the participants the some questions. What do you think will be the next outcome? Will the prior results affect following flips? How many of you think the coin will flip a head to balance the tails? How many think that the coin will continue to land on tail since the coin has been landing on tails in our previous flips? How many think it does not matter, the coin will either land head or tail. Are both outcomes equally likely? Does "head" have the same chance of occurring as "tail"? The idea that the past outcome influences future outcome, is a fallacy.

Model this activity for the students. Flip a coin 20 times and keep record of the flips.

For example TTHTTHHHHTTTHTTHHHTH is a string of outcomes for the 20 flips. We will examine the original string at least 10 different times and identify doubles: two heads or two tails in a row are doubles. We will record whether the flip following the double was "same" as in the double or "different" from the double.

TTHTTHHHHTTTHTTHHHTH

TTHTTHHHHTTTHTTHHHTH
d

TTHTTHHHHTTTHTTHHHTH
d

TTHTTHHHHTTTHTTHHHTH
s

TTHTTHHHHTTTHTTHHHTH
s

TTHTTHHHHTTTHTTHHHTH
d

TTHTTHHHHTTTHTTHHHTH
s

TTHTTHHHHTTTHTTHHHTH
d

TTHTTHHHHTTTHTTHHHTH
d
TTHTTHHHHTTTHTTHHHTH
s
TTHTTHHHHTTTHTTHHHTH
d

The above example shows 6 times a different outcome followed a double and 4 times a same outcome followed a double.

Activity Two (Small Group)

The students will work in pairs and keep a record. Flip a coin 20 times and record each flip as either H for Head or T for tails. Each student will have a string of 20 letters made up of H's and T's. Then start from the beginning of the list and look for doubles: Two heads in a row or two tails in a row. Record whether the flip following the double was the same as in the double or different. For example HHT will be recorded as different. HHH will be recorded as the same. Continue looking for the next set of doubles and count how many "same" and how many "different" you got. Note that in a string like TTTH, T that follows the first two T's, will be recorded as "same." The second and the third T also make a double TTTH. H following the second set of double will be recorded as "different." It should be the following string HHHHT provides 3 sets of doubles with 2 "same" and 1 "different."

Activity Three (Large Group)

Compile a class list of "same" and "different." The number of "same" and "different" will more or less be close to each other. Ask the class why this is the case. Enable the class to see that the answer is related to the idea of probability of heads or tails being ½. Ask the class if the probability of getting a head or tail is affected when you have a string of heads in a row. More importantly, why does the probability remain ½? Does the coin remember that it had flipped a head on its prior flip?

Name of Students	Number of Same s	Number of Different d
Teacher	4	6
Total	Sum Same s	Sun Different d

The belief that the coin remembers what it flipped earlier, the idea that the past outcome influences the future outcome, is a fallacy. How does this fallacy affect the rolling of a dice? Does rolling a 1 more likely in the event that number other than 1 was rolled?

The above experiment aimed to help students identify the fallacy. The class should become comfortable with the notion of independent events. Outcome of one event does not affect the outcome of another event. Ask the students to consider Juan's situation. Juan flips a coin and gets two heads. He flips again. What is the probability of another head?

Activity Four (Individual Work)

Another related activity and easy to do in a class is flipping a coin 20, 30 or 50 times. Keep a record of the number of flips. Students can use tally marks to keep count of the number of heads and tails. Students can flip 20, 30 or 50 coins together to save time. Students can then answer for themselves, what is the total number of heads and tails. Who in the class has the largest and the smallest number of heads? How many experiments will give exactly 10, 15 or 25 heads? Compile a class list.

Name of Students	Total Number of Heads	Total Number of Tails
Total	Sum of all the Heads	Sum of all the Tails

Hopefully, the rule of large numbers will come into play. The class data will show that the number of heads is close to the number of tails. Again, pose the question, why are the two number close? Relate the answer with the idea of probability tossing a head or a tail as being one out of two or half.

Following Lessons

Introduce the terms "theoretical probability" and "experimental probability" in following lessons. Theoretical probability is based on a model. When a coin is flipped, both head and tail are equally likely outcomes. Thus probability of heads or tails is one out of two ½. Observed probability is the result of experiments. It uses data based on observed outcomes when experiments are performed. For example, if 20 flips of a coin result in 12 heads and 7 tails, the observed or experimental probability of heads is 12/20= 0.6. It is more than ½.

Introduce simple probability questions along with related simple questions with "not." Example: If probability of raining is 3/5, what is the probability of not raining? Supplement the lesson with TAKS problems that require students to calculate the probabilities of independent, equally likely events. For example, the students can calculate the number of outcomes for a coin flip followed by rolling a die. They can also calculate the probability of getting a head followed by a number, say 3. Permutations and combinations can be introduced. An example of the Texas state lotto Pick 3 can be considered. Students can learn to answer permutation and combination type of questions using calculators.

As a final application project, students display the games they created along with the rules used to play the game. They can choose to display the graphs they created during the class work. As an alternative to game creation, students can do research in the computer lab. They can choose to focus on probability and statistics and how it is used in news and magazines. They can see how probability is applied in the real world to make decisions.

CONCLUSION

This unit is ambitious in that it hopes to spark the students' interest and keep them engaged in learning mathematics. In this unit, a number of ideas related with probability, such as equally likely events, independent events, theoretical and observed experimental probability, have been explored. These concepts are not easy to understand and are counter-intuitive even to adults. The unit enables students to explore these ideas by making conjectures following activities and recording data. Students then test these conjectures by analyzing the data generated. They use the data to calculate the statistics, and represent the data with graphs. They attempt to interpret and derive meaning from the data generated. Lastly, the students apply their understanding to solve TAKS type problems to show mastery of the concepts.

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