## Vectors in the Real World

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> When I was young, I said to God, 'God, tell me the mystery of the universe.' But God answered, 'that knowledge is for me alone.' So I said, 'God, tell me the mystery of the peanut.' Then God said, 'Well, George, that's more nearly your size.'
>
> \- George Washington Carver

## INTRODUCTION

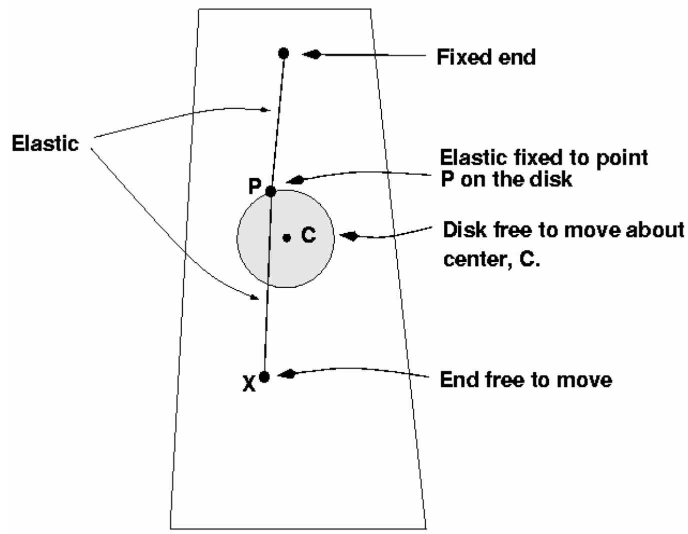
High school students always ask how people really use geometry and vectors. Well to define the unit vectors must be understood. Vectors describe the magnitude or amount of something, in this case force, and the direction it is pointing toward. Principles studied in geometry are used to break down real world examples to study. This unit will allow my students to explore the connection between math, science and real world applications. Physics uses mathematics as a language to explain the physical world around us. Math allows architects to design buildings, bridges, and homes by using the relationships developed in physics. This unit brings together Newton's laws of motion, geometry, and local architecture.

A goal that is always important is to show students the relevancy of the material they are studying. This unit is focusing on the architecture of ancient and present day. The students will have the chance to explore the different polyhedron used in building strong and beautiful shapes. We will begin studying Sir Isaac Newton's laws of motion, the first two laws, (Inertia and Force = Mass x Acceleration), are easy concepts for the students to grasp and apply to their everyday lives.

Working with my fellowship group I was has introduced me to so many exciting ways to look at math that is helping me to develop a curriculum from the information about patterns and geometry. It was such a totally different way of looking at the world around me that I had never even thought of. One example is Zeeman's Catastrophe machine.

This machine is based on the catastrophe theory discovered by René Thom. The Zeeman Catastrophe machine was developed by Dr. E.C. Zeeman. A mathematical catastrophe is a point in a model of an input-output system, where a very small change in the input can produce a large change in the output. An example would be putting a very small amount of weight on to structure that already has its maximum weight capacity. The machine helps to predict buckling in high stressed materials. So its practical applications are enormous in building structures. I realized that I could develop my curriculum studying the simplest of patterns that are used.

Zeeman Catastrophe Machine



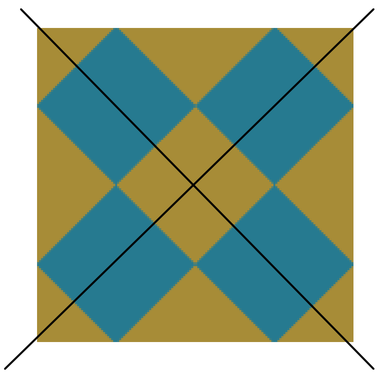

Investigate how P moves as we vary X – the control point.

## UNIT BACKGROUND

This unit will look at the different patterns and shapes that architects use to have a balance of forces. Starting with the most simple to some of the more complex, the students will discover that these shapes were not chosen by accident. This will bring to light the strong relationship of math and science. Once the relationship is shown we will go further and express what forces are acting on these structures using vectors. Based on the knowledge of Newton's Third Law of Motion the students will be able to predict and then prove if the building is in equilibrium. Bringing this idea to life will have to start with the simplest of patterns. Lattices are simple to build and provide quick vector representation. Starting with the simple patterns and shapes really helps the student to understand how more complex objects are made. Students will discover that architecture of any kind must be sound as well as pleasing to the eye. Students should be shown the beauty in strong shapes.

Symmetry will be the key focus of this unit. Symmetry is very important because the human body is symmetric and that is our first expression of symmetry. Even when students are drawing what they imagine they still have symmetry evening their doodles. Most structures in Houston have some type of symmetry. It is this symmetry that helps to catch the eye. It also adds to the overall aesthetics of the structure. The students will be able to identify what symmetries different structures have and develop free body diagrams of the forces acting on these structures. The students will be allowed to develop their own ideas on the relationship between geometry and physics. Digital images of different buildings and bridges around our campus will be taken by the students as part of a hands-on geometry hike. The students will then discern all of the support and load forces they think are acting on just one of the structures.

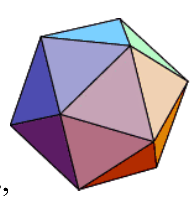
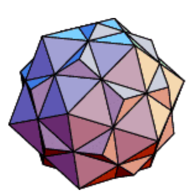
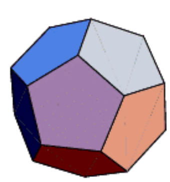
I plan on using the symmetry of ancient civilizations as examples of the union of art, geometry, and physics. Some examples can be found in Egyptian, Greek, and Chinese history. The simple shapes they use will be squares, triangles, and arcs. The students will see how these simple shapes are used to build complex patterns. It is interesting that these patterns are not accidental but have a definite mathematical model. An example of this is if we have two different sized squares and duplicate them a few times and place them side by side at specific angles we can develop an amazing pattern.

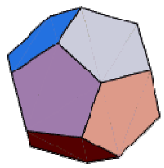
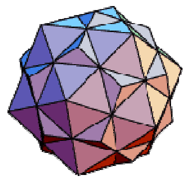
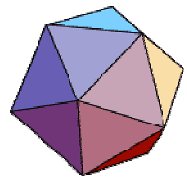


The pattern shown above is an example of taking a large square and fitting four squares of smaller areas inside then rotating the smaller squares by 45 degrees. This gives a pattern that has wonderful symmetries. When this pattern is folded on either line it is almost a perfect mirror image. Mathematicians, architects, and physicist study and use these shape phenomena to develop more elaborate patterns in everyday life.

The foundation of Newton's Third Law will be developed by using various resources such as textbooks, PowerPoint, and labs. The online resources I will use are websites that I have found in research. For example, Mathworld.com is where the students will see polyhedron of all types. The students will then build structures out of simple geometric shapes in repeating patterns and test the strength of their inventions. The simple shapes will consist of squares, rectangles, triangles, and other parallelograms. Then they will move up to pentagons, hexagons, and other two-dimensional polygons. The class will then be ready to move up two three-dimensional shapes to study for a moment The students will also build geometric shapes such as:

icosahedrons,

dodecahedrons,   

Implementing this unit will give the students a chance to answer the age old question: “Why do I need to learn this?” This is a question that has plagued teachers everywhere, and now the students will be able to answer that question on their own. Students will take all of the theoretical information on Newton’s laws of motion and synthesize them into a testable bridge. Two main points about this unit is that it is designed to include the visual, auditory, and tactile learner to all take ownership in the material. This unit is a student centered unit because the students can brainstorm and develop their ideas. They can also test and improve before presenting their final product.

The visual learner will get a chance to see the laws of motion in action. The unit will contain many pictures of buildings and bridges from Houston, TX and around the world. The visual learner will then be able to connect the theory with different forms of application. The Auditory learner will gain information from the reading of supporting text from the online physics text we use in connection with our lessons. The tactile learner will use hands-on applications to help foster a better conceptual understanding.

**Newton’s First Law of Motion**

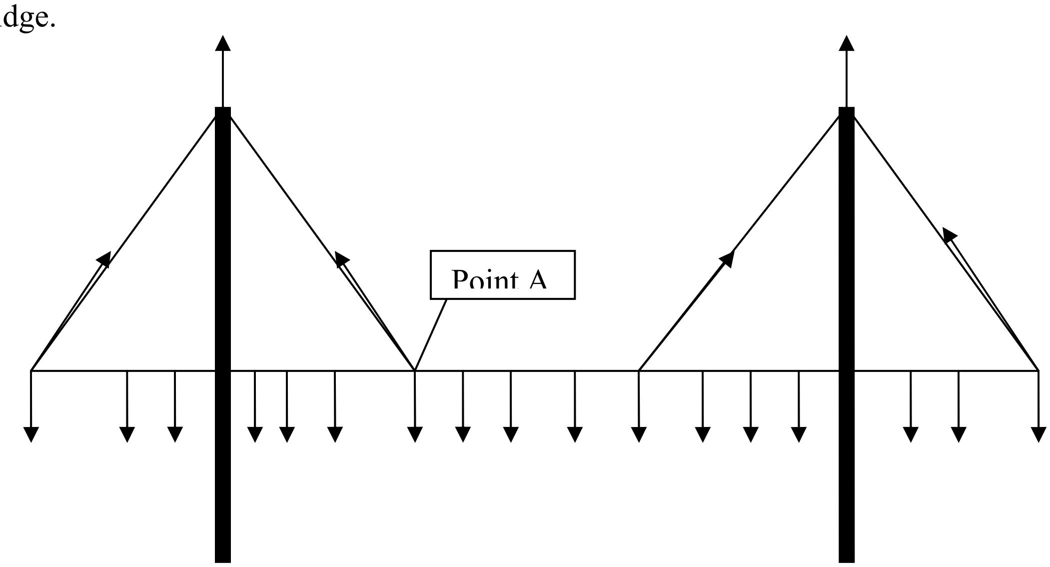
Inertia is an object’s tendency to be at rest or in motion until something, an unbalancing force, acts on the object. That is Newton’s First Law of Motion. One way to look at inertia is by trying to pull a sheet of paper from under a quarter. Inertia is what you feel when you are in a car and the brakes are applied hard while the car is moving at fifty miles an hour. As the car slows quickly your body is still moving at fifty miles an hour: that is why you pitch forward. Thank goodness for seat belts. Inertia is why we have air bags, to slow the acceleration of the body against the frame and interior of the car. All buildings feel the effect of inertia from their own mass being affected by the force of gravity.

**Newton’s Second Law of Motion**

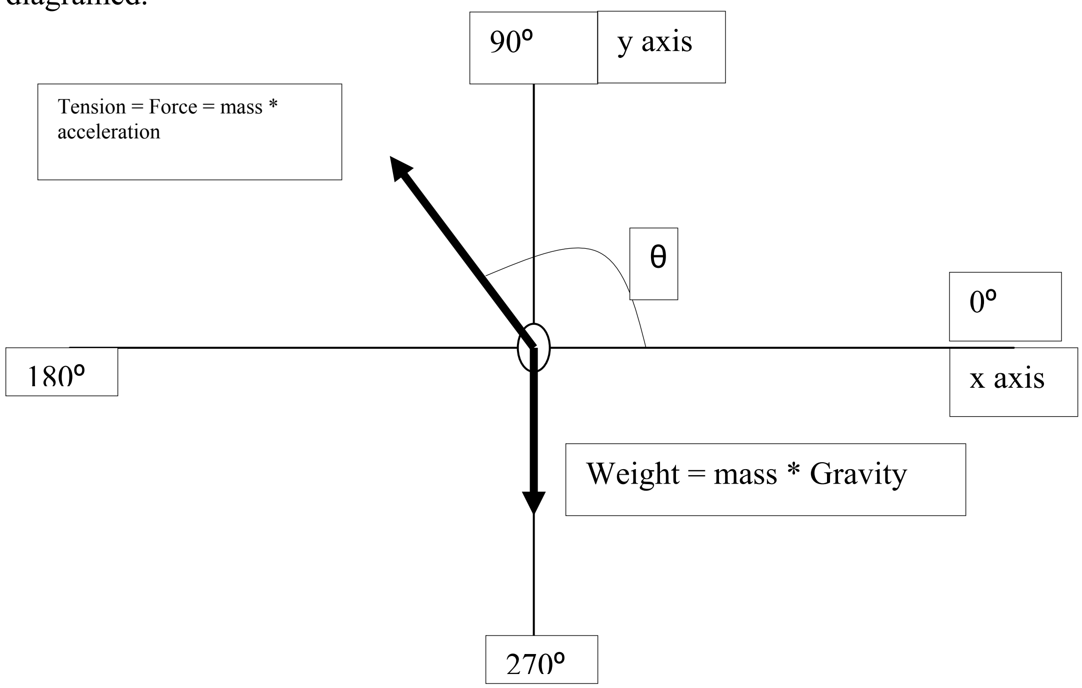
The Second Law defines what a force is. In order to understand what force is, you must understand acceleration. Acceleration is simply a change in velocity in measured amount of time. A force is an acceleration that is applied to a mass or object. Whatever it is that causes the change in velocity is the force. An example of force is very simple to show. A car will sit still on a level road even with the engine on. Only when the car is in gear and the accelerator is pressed down does the engine apply a force to the transmission causing the transmission to apply a force to the wheels does the car move. Force is something that we on earth are dealing with at all times. Our mass is constantly under the acceleration of gravity. Gravity is the force that pulls everything toward the center of the earth. A force can be a push or pull. A bridge, for instance, will have both pushes and pulls acting on it at one time. For the bridge to remain intact all of the forces must be balanced. How does a student represent forces that are balanced? The answer to this leads to Newton's Third Law and the focus of my unit.

**Newton's Third Law of Motion**

Newton's Third Law states: Whenever one object exerts a force on a second object, the second exerts an equal and opposite force on the first. This law is more commonly phrased as "to every action there is an equal and opposite reaction." An example to better explain this concept is standing on a wood beam. If the wood beam matches the amount of force your weight creates you remain standing. If your weight is more than what the beam can stand then board breaks and you fall. Determining weight distribution allows for the development of better bridges. This leads into the balancing of forces. The Third Law that deals with equilibrium of forces is very hard for the students to grasp. I will use examples of geometric designs to show how forces need to be balanced. The students will be able to see that when everyday forces like the force of gravity, the force of wind, and friction, are balanced, then buildings and other structures are safe for the population. In order to correctly determine the magnitude and direction of forces involved, physicists use free body diagrams. What exactly is a free body diagram? It is a diagram that simplifies a system of different real world situations into graphical representations. The focus of this unit is Newton's Third Law of Motion and the bridge will be the main ingredient. Here is an example drawing the arrows representing the different forces acting on this bridge.



The diagram above shows very simply the forces acting on the structure. The arrows pointing down represent the force due to the acceleration of gravity. The arrows pointing up represent tension from the support wires. Trying to find the equilibrium of these forces all at one time is daunting enough for college seniors, so we will concentrate on very small areas. First we focus on the areas closest to the supports. Instead of drawing the whole area, we use the free body diagram. A free body diagram is a graphical representation of the forces acting on an object. In this case the object will be the connection point on one of the joints of the bridge. Point A is the joint that will be diagramed.




As shown, the free body diagram simplifies the system into two main forces. The next step is to express the forces in terms of vectors. Vectors have magnitude and direction. We express the magnitude of force in newtons and the direction in degrees. All of this is on a Cartesian coordinate system. The directions are based on the positive x axis with the origin at 0 degrees. The vectors then can be resolved into x and y components. How do you do that? We have to use the sine and cosine functions to find the x and y components of each force and then add up the x components and the y components and find the resultant magnitude and direction using the inverse tangent. It is just that simple. This is what all looks like symbolically.

x-component

tension on the x axis = $T_x = T * \cos(\theta)$

y-component

tension on the y axis = $T_y = T * \sin(\theta)$

Weight = mass * gravity = $W * \sin(90°)$

Resultant Force

Magnitude $F = \sqrt{( T_x^2 + (T_y+W)^2 )}$

Direction $\theta = \tan^{-1}( (T_y+W)/ T_x )$

These two examples above are simple but that is the point even when making bridges. It will do the student no good if the work becomes to cumbersome. Using basic force vector representation the concept of Newton's laws of motion can be seen more clearly.

This unit is based on the CLEAR Objective PHYS 12.5: The student will be able to demonstrate the effects of forces on the motion of objects. Using pictures, taped demonstrations, projects and laboratory experiments, I envision a multimedia unit that the physics department at DeBakey HSHP can use. I also want the unit to be flexible enough that it can be adapted and used as review. I am excited that working with University of Houston faculty will bring this unit to life.

The students here at DeBakey are all magnet level and above. They are competitive and always looking for challenges. This unit will be a tough and exciting one to keep their interest peaked. The first step of the unit is to begin by immediately showing my students the basic geometric shapes used in the buildings, bridges, and other structures around us. An example of basic geometric shapes in architecture is the sphere or in this case part of a sphere known as a dome. A dome is unique because it is a beautiful piece of architecture that is very strong and resilient. There is one example of a dome here in Houston I am using for this unit and that is at the Reliant Center. Using pictures from different angles students will discover for themselves why a dome was used instead of a flat surface. Then I will have the students develop free body diagrams of the forces acting on these different shapes. Even though the students will be studying rounded and flat surfaces, the vectors will be expressed as simply as possible in Cartesian coordinates. This will definitely challenge them to think outside of the textbook. I see this unit having a field trip component where the students will study the different building styles in downtown Houston.

Lesson one of this unit will serve as an introduction to Newton's Third Law of Motion. The purpose of starting the unit with Newton's law is to serve as the foundation of this unit. No matter what kind of activities are done, they should all be connected with or be able to point to an equilibrium of forces. The lesson will start with the definition of equilibrium. Students will then determine when objects are in equilibrium. This determination will not made by sight, but learning how to make free body diagrams. Free body diagrams allow the students to strip away all of the extraneous information and focus on just the forces involved. Students can then list the magnitudes of the forces and their directions. The main outcome I want is for the students to be comfortable with simple examples first, for instance a rope attached to a hook supporting a weight. By then end of this lesson they will be doing free body diagrams for a host of different models. One of those types of models will be a bridge.

Fig. 1: Erskine bridge in Scotland (Field).

Bridges are very good because they are update applications of an old law of motion. First there are two things that vectors describe, that is the magnitude of something and which direction the force is going in. After vectors have been defined, how do students use them to interpret the information?

## LESSON PLANS

### Lesson Plan 1: What Goes UP…

The unit will start with definition and discussion of what Sir Isaac Newton's Third Law of Motion really means.

***Objectives***

- Students will analyze examples of uniform and accelerated motion including linear, projectile, and circular. (TEKS 112.47.c.4.A)
- Student will develop and interpret free body diagrams. (TEKS 112.47.c.4.D)

***Materials Needed***

Paper
Coins
Gravity
Toy Car

***Procedure***

The lesson will open with demonstrations of inertia and force. The inertia demonstration will have the students pull slips of paper from under a coin. The demonstration of force will have one student pushing a student in a chair. The students will then explain how the velocity changed from a stand still to moving forward. The students will discuss the meaning of acceleration.

The objective of the first lesson will be for the students to determine how the Third Law of Motion can be explained in everyday situations. For instance have the students face each other in pairs and extend their hands until they meet. I would have student # 1 in the pair lean forward toward student #2. Student #2 will have to apply a force to keep student #1 from falling. By holding student #1 steady, student #2 has created equilibrium of forces. The students will have a hands-on example of Newton's Third Law.

The guided practice would consist of having the two student teams draw sketches of what they did in their demo. The students would then in their own words explain what would have happened if student #1's force was much greater than student #2's or vice versa. I would then ask the question: "What exerts the force on a car?" Response: "A common answer is that the engine makes the car move forward. The engine makes the wheels go around. But what good is that if they are on slick ice or mud? They just spin. A car moves forward due to the friction force exerted by the ground on the tires, and this force is the reaction to the force exerted on the ground by the tires."

Independent Practice will have the students find and examples of Newton's Third Law complete with pictures either drawn or photographed with detailed explanations. The explanations will include why the student picked a particular example, and defend the selection with sound principle.

The Assessment phase will have the students go out around their neighborhood with a camera to take pictures of interesting buildings or homes and breakdown the forces that are acting on each picture. The student should have a minimum of five pictures for the portfolio. The end of this lesson will lead into the beginning of how repeating certain patterns increase the strength of an object like a building or a bridge.

**LESSON 2: How We Use Newton's Third Law**

***Objective***

- Students will demonstrate the effects of forces on the motion of objects. (TEKS 112.47.c.4.C)

***Materials Needed***

K'NEX® Bridge set
Corkboard
Rubber bands, roughly 5-8 cm long,
Thumbtacks, at least 2 of them with "flat tops"
Pencil eraser
A 5 cm by 5 cm square of cardboard
Ruler

***Procedure***

The review of this lesson will be on the portfolios the students turned in. We will examine each one for repeating patterns. Before we look for repeating patterns, the students will develop a background on simple patterns. The students will view a presentation on shapes and patterns. The presentation will give students the website addresses of Dr. Mike Field and Dr. George Hart.

The students, along with the teacher, will create and discuss polyhedron of different shapes. This will make the connection between geometry and physics. Geometry is the study of the relationships between lines and angles. In order to use vectors correctly, students will discover how vectors are broken down into components. Since we will be working in the Cartesian coordinate system, that means x and y components. Once that connection has been established, the students will be able make models of the buildings around them.

This is a hands-on unit all of the way. It has to be because the concepts are hard to grasp. The students will be using building materials called K'NEX. It is a comprehensive bridge set. Although it is designed for the younger students, like many toys it is based on sound physics. This kit will allow students to explore the basics of bridges. They will also have hands-on understanding on the evolution of the structure of bridges and how repeating patterns make a bridge stronger.

Before the building phase of this lesson I will show the students pictures of the bridges over Southwest Freeway 59. These bridges are simple but strong in their design. We will dissect these pictures by way of free body diagrams. The students' free body diagrams will be extended into plans that will help them design a bridge for testing. The students will be put in teams that will do different test involving stress and strain. The student will record how much weight it will take to buckle a 10 centimeter long piece of model making wood. This will be the time the Zeeman's catastrophe machine will be introduced. The students will first going to build the machine by following these instructions:

1. Gather
2. Attach the background grid to the corkboard with thumbtacks, so that the rectangular grid area is to the right.
3. Draw diagonal lines on the square of cardboard, and on one of these lines at a point 2.5cm form the center, stick a "flat top" thumbtack through the cardboard and secure it with the pencil eraser.
4. Mount the square of cardboard on the background grid using a "flat top" thumbtack. The thumbtack should pass through the center of the square of cardboard and through the origin of the coordinate system marked on the background grid. The square should be able to rotate freely.
5. Place a rubber band around the shaft of the thumbtack having the pencil eraser. With another thumbtack, attach the other end of the rubber band to a point on the negative x-axis. The point should be selected so that the rubber band is stretched at all angles of rotation of the square of cardboard.
6. Place a second rubber band around shaft of the thumbtack having the pencil eraser. With another thumbtack, attach the other end of the rubber band to point x=5 on the

positive x axis. Rotate the square of cardboard and determine the angles of rotation at which the square is in equilibrium. Mark the angles on the background grid, and label them each "5"

7. Repeat the previous step, but with the end of the second rubber band attached to the points x=6, x=7, x=8, x=9, x=10, x=11, x=12, x=13, x=14
8. Plot the equilibrium angles found, against the x coordinates of the endpoint of the second rubber band, on the rectangular grid. Let the first horizontal line above the x-axis represent theta=0.1 radians, the second theta=0.2 radians etc. Note that angles around the semicircle are marked every 0.1 radian.
9. Label the solution branches on the diagram "s" or "u" according to the stability of the equilibrium solutions.
10. Does the figure on the right remind you of any familiar tool that might be used on a farm?

The students will use this machine to estimate breaking points. The class will see an example of a bridge that is not built correctly to handle the forces that act on it. The bridge that collapsed in the 1950's in the state of Washington was caught on film. We use the Annenberg film series on resonance to study this phenomenon.

**Lesson 3: Built to Last**

***Objective***

- Students will demonstrate the effects of forces on the motion of objects. (TEKS 112.47.c.4.C)

***Materials Needed***

Sketches of models to build
String
Assorted Masses from .5 to 1 kilogram
Two chairs to lay the bridges across

***Procedure***

Lesson three will focus on team development of a finely engineered arch bridge. The students will develop accurate predictions of the weight limit of their design based on the free body diagrams and equilibrium of the forces they calculate. These sketches will be very detailed in showing the supports of the bridges. The sketches or blueprints will be drawn in a one to one scale in centimeters. This lesson should take approximately two ninety-minute blocks. The students will present their plans and calculations for evaluation. This requires that the instructor must very comfortable with balancing forces. The students will need to meet in their groups to correct any errors in their plans before constructing their bridges. That is where the K'NEX Bridge set comes into play. The junior engineers will put their plans to the test. The students in their groups will work to implement the design they have created. This will allow them to see how careful planning and development will help them build wonderful and amazing things. The unit’s big finale will consist of a small contest to see how much force each bridge can support. Each group will bring their finished bridge up to the testing area which will be two lab tables with a space in between. The bridge will be placed over this space and a string with a mass hook will be attached to the middle of the bridge. The teacher will supervise as the students begin add weight the mass hook. The starting mass that we start with will be half a kilogram. The mass will be increased be half a kilogram each time. So the bridges will have to be strong to withstand the weight these masses will apply to it. There is a misconception about mass and weight that will be needed to dispel. Mass is the amount of a material present. Weight is the actual force the earth applies on the mass by the acceleration due to gravity. The SI unit for force is Newton. The winners are the groups whose bridge can support 15 or more newtons the students will convert that into kilograms by dividing 15 by 9.81 $m/s^2$.

## CONCLUSION

An important idea to remember for any unit is that it must make sense to the student if there is going to be any real learning to take place. A teacher must build a bridge from what the student has already learned to a new and more complex concept. Teachers must allow the students to be hands-on and exploratory. Let them know that in the real world you will have to continue building and rebuilding until you get it right. This unit will require that the teacher go through and build some of these things first to see where the pitfalls and questions will come from. Vectors describe things that are in the real world and now the student will be able to answer that question of why do we study this. We study physics because it helps to answer just some of the mysteries of the world around us. Once we have answers of how forces work on earth, we use the information to build improvements on our own personal abilities.

There is definite and measurable improvement in students’ understanding of physics concepts when they are allowed to imagine, design, and build their own creations. This unit allows the student to get a taste of combining theory and application. When they build their bridges they are bringing a dead textbook to life.

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